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Estimating

DEPENDABLE ANNUAL STREAMFLOW

in the Unglaciated Allegheny Plateau

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Growth Through Agricultural Progress

ESTIMATING DEPENDABLE ANNUAL STREAMFLOW IN THE UNGLACIATED ALLEGHENY PLATEAU 1/

D. L. Brakensiek 2/

The B15 Problem Area in Soil Conservation can be viewed essentially as the Appalachian Plateau Physiographic Province. Fenneman (3) 3/ indicates that this Province for systematic study should be divided into seven areas. These differ in the character of their underlying rocks, in altitude, in degree of dissection, and in the presence or absence of glaciation. Fenneman's division was partly verified by Morisawa (4) through the use of quantitative morphological parameters. It was found that the Allegheny Mountain, Unglaciated Allegheny Plateau, and Cumberland Plateau Divisions showed significant differences in certain morphological parameters.

The study reported here is limited to the largest and most typical division of the province, the Unglaciated Allegheny Plateau Division. Watersheds utilized in this study further define the area of study and future application of results (see Appendix Figures 1 to 4). Geographically the study area covers southeastern Ohio, southwestern Pennsylvania, western West Virginia, and a portion of eastern Kentucky.

Morisawa (4) describes the Unglaciated Allegheny Plateau Division as mature in stage, with fine texture, and moderate to great relief. Both surface elevation and relief are lowest on the western edge in Ohio and Kentucky where elevation averages 1,200 to 1,300 feet with relief of 200 to 300 feet. Eastward surface elevations reach 1,500 to 1,600 feet. In West Virginia valleys are steep-walled and deep, with an average relief of 1,000 feet. The surface of the division is characteristically rugged, being mainly in valley side slopes. Divides are long and linear with many intermittent areas of rolling upland. Valley flats are generally narrow and confined to larger streams and lower major tributaries. Stream valleys are generally narrow, steep-sided and often deeply incised. However, many become gentle in gradient near their heads with broadly rounded divides.

BASIC DATA

Annual streamflow amounts, in units of inches over the watershed, for watersheds ranging in size from under 100 acres to 400 square miles were tabulated on a calendar year basis. For consistency the calendar year was

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3/ Numbers in parentheses refer to Literature Cited at end of this report.

used with tabulated U. S. Weather Bureau annual precipitation data. Agricultural Research Service watersheds at Coshocton, Ohio, and U. S. Geological Survey gaged watersheds were utilized. A list of watersheds is given in Column 1 of Table 1.

According to Climate of the States (9) long-time mean annual precipitation in the study area probably varies from 36 to 48 inches. However, since tabulations of annual streamflow data were often for different periods of time, the geographic precipitation difference would be further accentuated. Thus to correct for this annual watershed precipitation was computed for use as a concomitant factor.

Annual precipitation amounts in inches for appropriate stations were tabulated from the U. S. Weather Bureau's "Climatic Summary of the United States -- Supplement for 1931 through 1952" (7). For later years the annual summaries of State Climatological Data (8) were utilized.

Iso-lines of annual precipitation were constructed covering the pertinent sections of each state for each year of record. From the iso-line maps annual values of watershed precipitation were determined for each watershed, and thus from these average annual watersheds, precipitation was calculated for each watershed's length of record. This procedure was thought appropriate because very seldom was there one or more raingages actually within a watershed. Column 3 of Table 1 indicates that calculated watershed precipitation varied from 37 to 57 inches. This variation reflects geographic as well as length of record differences.

Obtaining detailed and consistent watershed physiographic parameters was not possible because different watersheds were covered by different scaled topographic maps, 1/62,500; 1/24,000; or 1/4,800. However, the following simple items were obtained: watershed size in square miles, perimeter in miles, length of principle water course in miles, and maximum relief in feet. More detailed physiographic factors would have required a field survey.

The exact location of the gage site for each watershed, Column 1, Table 1, can be found in the U. S. Geological Survey Water Supply Papers for the Ohio River Basin.

FORMULATION OF STUDY

The purpose of this study is to develop a procedure for estimating annual streamflow for any level of dependability. Assuming that annual streamflow amounts are statistical variates, we can write a formal expression (1) for estimating probability - magnitudes as

$$Q_t = \overline{Q}\,(1 + CV_q\ K) \qquad (1)$$

where for a particular watershed

Q_t = annual streamflow that will not be equalled for a return period T

$\bar{Q}$ = average annual streamflow

CV_q = parameter reflecting inherent variability of annual streamflow

K = standardized deviate for a probability level of (1/T); its value also depends on the statistical distribution of the variates annual streamflow

The application of equation (1) to a particular watershed requires that three items be known or procedures for estimating them be available. The items are:

(1) An appropriate statistical distribution for annual streamflow.

(2) $\bar{Q}$, average annual streamflow.

(3) CV_q , inherent variability parameter.

These will be considered in the next sections.

Statistical Distribution

No clear-cut statement can be made about the statistical distribution most appropriate for annual streamflow amounts. The choice in many hydrologic studies is often made by trying several and selecting the one that most consistently fits the available sets of data. This study is no exception in that the normal distribution, log-normal distribution, and gamma distribution were all investigated.

Annual streamflow data were plotted on normal and log-normal probability paper to test whether one or the other would be appropriate. A graphical test of the adequacy of the gamma distribution is not a simple task as the distribution cannot be standardized for construction of a "universal" probability paper. However, several test graphs were prepared for this distribution.

The plot of each set of watershed data on normal probability graph paper indicated that this statistical distribution consistently described the data. (See Appendix Figures 5 to 41). A discussion of the graphical scheme is presented in Appendix 1.

There is probably some theoretical justification for a normal distribution describing the accumulated probability function of annual streamflow in this study area. One can consider that annual streamflow is composed of the sum of many independent streamflow events occurring during the year. According to the central limit theorem then this sum, annual streamflow, under very general restrictions (2) would be asymptotically normally distributed. For an area of low annual streamflow, where very few independent events occur in a year, annual streamflow would be skewed considerably from a normal distribution.

Table 1.--Watershed Information.

NAME	$\bar{Q}$	$\bar{P}$	CV_Q	CV_P	Size	Length of record
OHIO	Ins.	Ins.			Sq. mi.	Yrs.
Short Creek	14.49	41.0	0.3356	0.1606	124.0	15
Lisbon Creek	13.57	38.6	--	--	6.08	10
Mill Creek	13.61	37.79	.3997	.1476	27.5	20
Salt Creek	13.40	39.4	--	--	10.6	10
Tar Hollow Creek	11.60	40.6	--	--	1.50	10
Home Creek	11.15	37.2	.4208	.1574	1.64	20
Yellow Creek	15.42	39.8	.3262	.1443	148.0	16
Coshocton, No. 97	12.3	37.4	.3783	.1474	7.16	20
Coshocton, No. 95	12.0	37.5	.3854	.1512	4.02	19
Coshocton, No. 94	12.2	37.5	.3734	.1512	2.37	19
Coshocton, No. 92	12.3	37.5	.3782	.1512	1.44	19
Coshocton, No. 5	11.37	37.5	.3975	.1512	0.545	19
Coshocton, No. 10	9.61	37.5	.4040	.1512	.191	19
Coshocton, No. 196	14.65	37.8	.3801	.1532	.473	20
Coshocton, No. 183	10.57	36.3	.4517	.1648	.116	19
Coshocton, No. 177	7.67	36.5	.4670	.1561	.118	18
Coshocton, No. 172	11.58	36.5	.4338	.1561	.068	18
WEST VIRGINIA						
Buffalo Creek	20.6	46.8	.2988	.1419	115.0	24
West Fork River	22.38	48.1	.3246	.1582	181.0	26
Skin Creek	22.46	49.0	--	--	25.7	11
Buckhannon River	28.6	57.2	.2587	.1423	277.0	26
West Fork River	23.5	49.1	--	--	102.0	10
Wheeling Creek	15.76	41.2	.3911	.1531	282.0	16
Little Kanawha River	24.38	51.4	.3146	.1430	155.0	19
Do	21.7	48.1	.3237	.1595	386.0	26
Coal River	17.16	45.5	.2884	.1421	393.0	26
Little Coal River	16.93	44.5	.3198	.1432	270.0	26
Steer Creek	17.63	45.8	.3306	.1548	166.0	19
Meadow River	24.91	48.2	.2134	.1367	287.0	26
PENNSYLVANIA						
Connoquenessing Creek	18.12	39.0	.2867	.1401	356.0	25
Crooked Creek	20.80	44.5	.2990	.1458	191.0	19
Dunkard Creek	17.14	43.4	.3309	.1501	229.0	16
South Fork Ten Mile	15.62	41.16	.3356	.1566	180.0	25
KENTUCKY						
Tygarts Creek	17.03	42.59	.4546	.1675	241.0	17
Johns Creek	16.3	44.4	.3749	.1634	55.7	15
Triplett Creek	21.9	45.2	.3800	.1662	47.5	16
Poor Fork	23.1	48.4	.3006	.1240	82.1	16
Little Sandy	16.4	43.6	.4400	.1989	398	19
Levisa Fork	15.1	43.3	.3364	.1598	386	18
Cumberland River	23.7	49.2	.2820	.1202	374	16
Blaine Creek	15.2	43.2	.4600	.2122	217	18
Licking River	15.7	45.5	.3997	.1814	140	19

Theoretically the gamma distribution with zero skew would also have fitted the data. H. C. S. Thom (5, 6) gives maximum likelihood estimators for the gamma distribution. The data of Appendix Figure 6, when substituted in these estimating equations, gave a skewness statistic of about 0.76. This is not zero, but if the confidence limits were known, they might encompass zero. However, the point is that for this set of data the gamma distribution did not reduce to the normal distribution which seemingly fits the data quite well. Even if it had, the simplicity of the normal distribution would make it preferable in this study to the gamma distribution.

Average streamflow, $\bar{Q}$

In an earlier section it was mentioned that average annual watershed precipitation varied from watershed to watershed because of geographical location and/or different lengths of record. Since part of the variation of average annual streamflow from watershed to watershed is certainly due to the precipitation difference, average annual watershed precipitation is used as a concomitant factor.

Several watershed physiographic factors were also studied as possible independent variables in an estimation equation for $\bar{Q}$. Three factors were investigated: watershed size, perimeter, and relief ratio.

It was found that watershed size or perimeter in combination with average watershed precipitation were significant factors. The relief ratio was not found to be a significant factor. Since size is a more common measurement than perimeter it was selected for the final estimating equation. The predicting equation is as follows:

$$\bar{Q} = 0.01334\,(\bar{P})^{1.915}\,(A)^{0.026} \qquad (2)$$

where

$\bar{Q}$ = average annual streamflow, inches

$\bar{P}$ = average annual watershed precipitation, inches

A = watershed size in square miles

The coefficient of determination associated with the logarithmic transformation of (2) is $R^2 = 0.86$. The standard deviation for each exponent,

1.915 ± 0.035

and 0.026 ± 0.001

indicates that the factors are quite highly significant in the statistical sense. In a later example computation it will be shown that the seemingly small exponent of the area term is also of considerable practical significance.

Streamflow variability, CV_q

The variability factor, CV_q, can be estimated for each watershed by applying least squares procedures to equation (1), i. e.

$$(CV_q) = \frac{\sum (Q / \bar{Q}) K}{\sum K^2}$$

This is possible because the K-values are easily determined from a Table of Areas for the normal distribution. Accumulated probability values are determined from the plotting position formula

$$m / (n + 1)$$

where

m = order number after items are ranked, large to small

n = length of record

Equation (1) was only fitted to sets of data of at least 15 years of length. It was felt that reliable accumulated probability functions could not be established for shorter records. The fitted lines are also indicated in Appendix Figures 5 to 41.

The calculated CV_q values were related to easily obtained factors. The best and most simple predicting equation found is as follows:

$$CV_q = 0.2340 - 0.0071 (\bar{Q}) + 1.5750 (CV_p) \qquad (3)$$

where

CV_q = parameter in equation (1)

$\bar{Q}$ = average annual streamflow, inches

CV_p = coefficient of variation associated with an appropriate record of annual precipitation.

The coefficient of determination associated with equation (3) is $R^2 = 0.71$. Both $\bar{Q}$ and CV_p are at least highly significant factors. CV_p would be calculated from an appropriate annual watershed precipitation record by the usual formula,

$$CV_p = \frac{\sqrt{\sum (P - P)^2 / (N - 1)}}{\sum (P) / N}$$

In terms of uncorrected sums of squares it could be calculated by the following equation:

$$CV_p = \sqrt{\frac{N}{N-1}} \sqrt{\frac{N \sum P^2 - (\sum P)^2}{(\sum P)^2}}$$

$\bar{Q}$ could be estimated by equation (2).

DISCUSSION

Equations (2) and (3) together with the assumption of normality completely define the probability-magnitude estimation equation for annual streamflow, equation (1).

Equation (2) indicates that average annual streamflow is almost related to the square of annual precipitation. Even though the exponent of the area factor is small it reflects a significant trend, that is, average streamflow increases with increasing watershed size.

Equation (3) points out some interesting items about year to year variability of annual streamflow. Streams that have higher average annual flows exhibit less year to year variations in flow. Also, watersheds that experience less variation in annual precipitation also have less variation in annual streamflow.

The following numerical example illustrates the application of the results derived in this study:

GIVEN:

(a) Watershed area $A = 356$ square miles

(b) Average annual watershed precipitation $\bar{P} = 39$ inches (see Table 2).

(c) $CV_p = 0.14$ (calculation shown in Table 2).

COMPUTATIONS:

(a) From equation (2)

$$\bar{Q} = 0.01334\ (39)^{1.915}\ (356)^{0.026}$$

$$\bar{Q} = 17.25 \text{ inches}$$

If the area term, raised to the 0.026 power, had been neglected $\bar{Q}$ would have been 14.81 inches. This would have been an underestimate of over 15 billion gallons of water!

(b) From equation (3)

$$CV_p = 0.2340 - (0.0071)(17.25) + 1.5750\ (0.14)$$

$$CV_p = 0.3320$$

Thus, the estimating equation for dependable annual streamflow for this example watershed is

$$Q_t = 17.25\ (1 + 0.33K)$$

The above equation can be used in one of several ways.

Table 2.--Precipitation Record for Connoquenessing Creek, Hazen, Pa.

YEAR	P 1/	COMPUTATIONS
	In.	
1932	35	
33	34	$\sum P = 974$
34	32	
35	39	$\sum P^2 = 38634$
36	40	
37	48	$N = 25$
38	37	
39	36	
1940	42	$\bar{P} = \sum P/N = 974/25$
41	36	
42	44	$\bar{P} = 39.0$
43	35	
44	37	$CV_P = \sqrt{\frac{25}{24}} \times \sqrt{\frac{25(38634)-(974)^2}{(974)^2}}$
45	46	
46	32	
47	36	
48	41	
49	33	$CV_P = (1.02) \times \sqrt{(0.018103)}$
1950	46	
51	41	$CV_P = 0.14$
52	42	
53	33	
54	44	
55	34	
1956	51	

1/ Calculated from annual iso-precipitation maps.

Table 3.--Selected Return Periods and Associated Frequency Factors.

Prob. of a lesser value	Return period, T	Frequency factor, K
Percent	Year	
95	1.05	1.64
90	1.11	1.28
85	1.18	1.04
80	1.25	.84
75	1.33	.67
70	1.43	.52
65	1.54	.39
60	1.67	.25
55	1.82	.13
50	2	0
45	2.22	- .13
40	2.50	- .25
35	2.86	- .39
30	3.33	- .52
25	4	- .67
20	5	- .84
15	6.67	-1.04
10	10	-1.28
5	20	-1.64
2	50	-2.05
1	100	-2.33

Using the values of K found in Table 3, values of annual watershed streamflow can be calculated for selected return periods. On the other hand, it may be preferable to plot the equation itself on normal probability paper and read values for any return period. Figure 42 shows the above derived equation plotted on normal probability paper. Two points needed to orient this line can be calculated as follows:

(1) One point should always be taken at Probability = 50% since for this value K = 0 and $(Q/\bar{Q}) = 1$.

(2) The second point could be calculated for an accumulated probability value of 95 percent. From Table 2 the value of K for this probability level is K = 1.64. Thus, the ratio

$$(Q_{95\%}) / \bar{Q}) = 1 + 0.33\ (1.64) = 1.52$$

The second point then has coordinates

(95%, 1.52).

Of course, any value read from a graph such as Appendix Figure 42 would be

multiplied by the mean to obtain the actual Q value. In the example, the values read from the ordinate scale would be multiplied by $\overline{Q} = 17.25$.

For a specific application it might be feasible to read an average annual watershed precipitation value from Appendix Figures 1 to 4. However, actual precipitation station records should be utilized for calculating the coefficient of variation of watershed annual precipitation.

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APPENDIX

Any graphical test for the "goodness of fit" of a theoretical accumulated probability function to observed data reduces to a plotting of the observed reduced variate to the theoretical reduced variate. That is to say the quantity

$$(x - \mu)/\sigma$$

is plotted versus the quantity

$$(y - \bar{y})/ s_y$$

Where the former is the theoretical reduced variate, in this study read from a table of areas of the normal distribution, and the latter the observed reduced variate. The condition of equivalence leads to the following

$$(y - \bar{y})/ s_y = (x - \mu)/\sigma$$

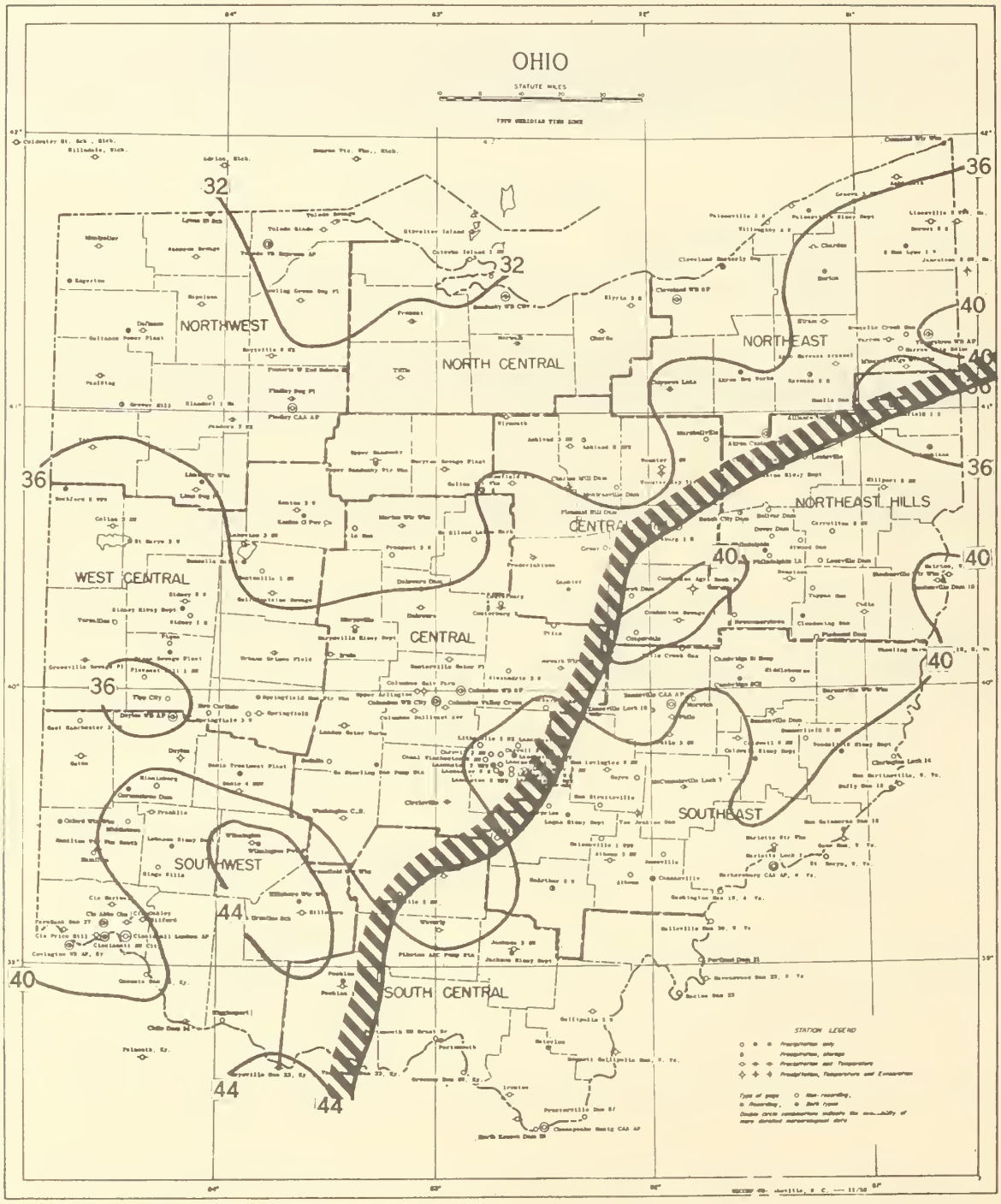
which reduces to $y / \bar{y} = 1 + (s_y / \bar{y})(x - \mu)/\sigma$

or

$$(y / \bar{y}) = 1 + (CV)K.$$

This is the same as equation (1).

Mean Annual Precipitation, Inches

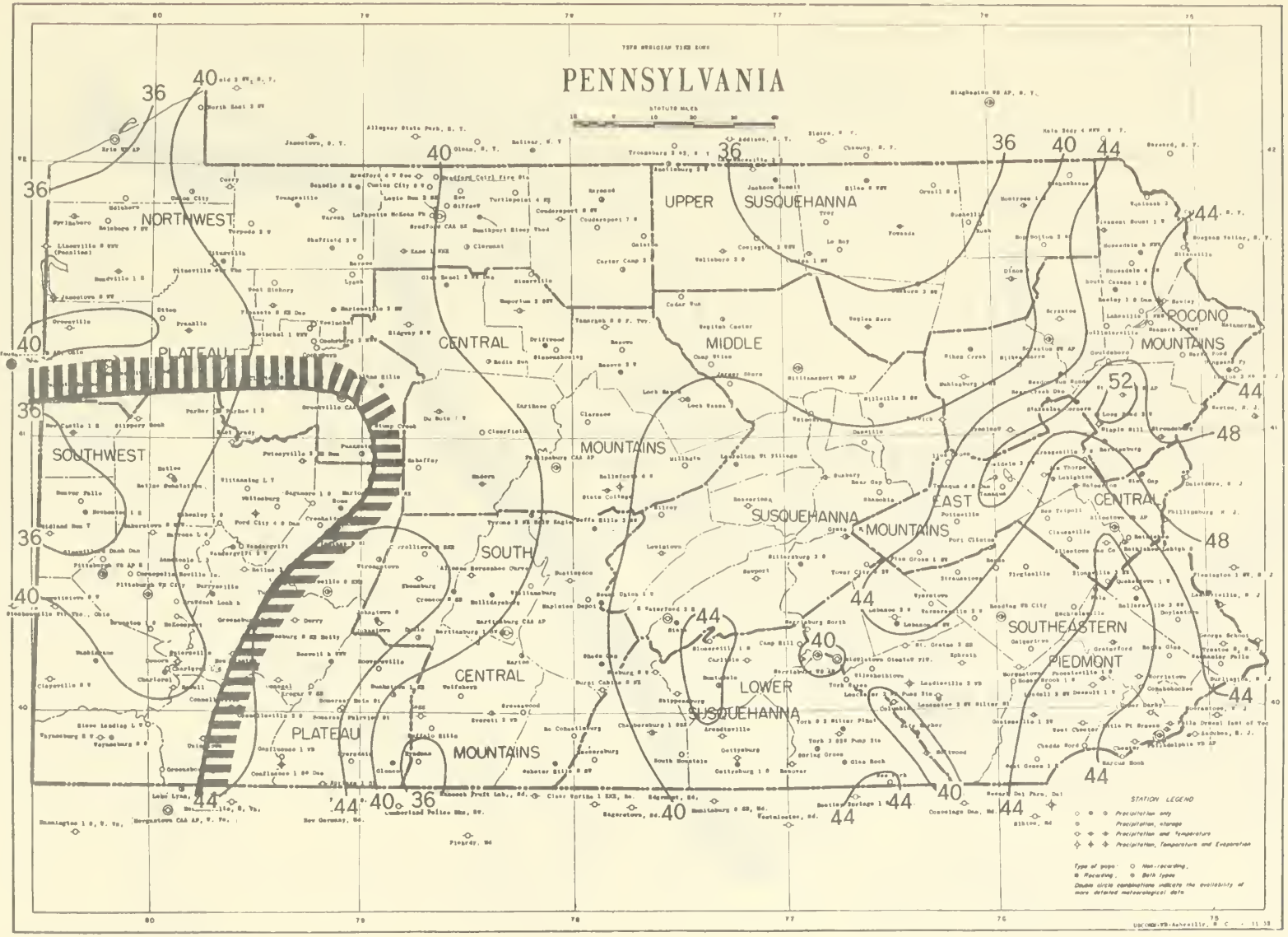


Based on period 1931-55

Isolines are drawn through points of approximately equal value. Caution should be used in interpolating on these maps, particularly in mountainous areas.

Figure 1.

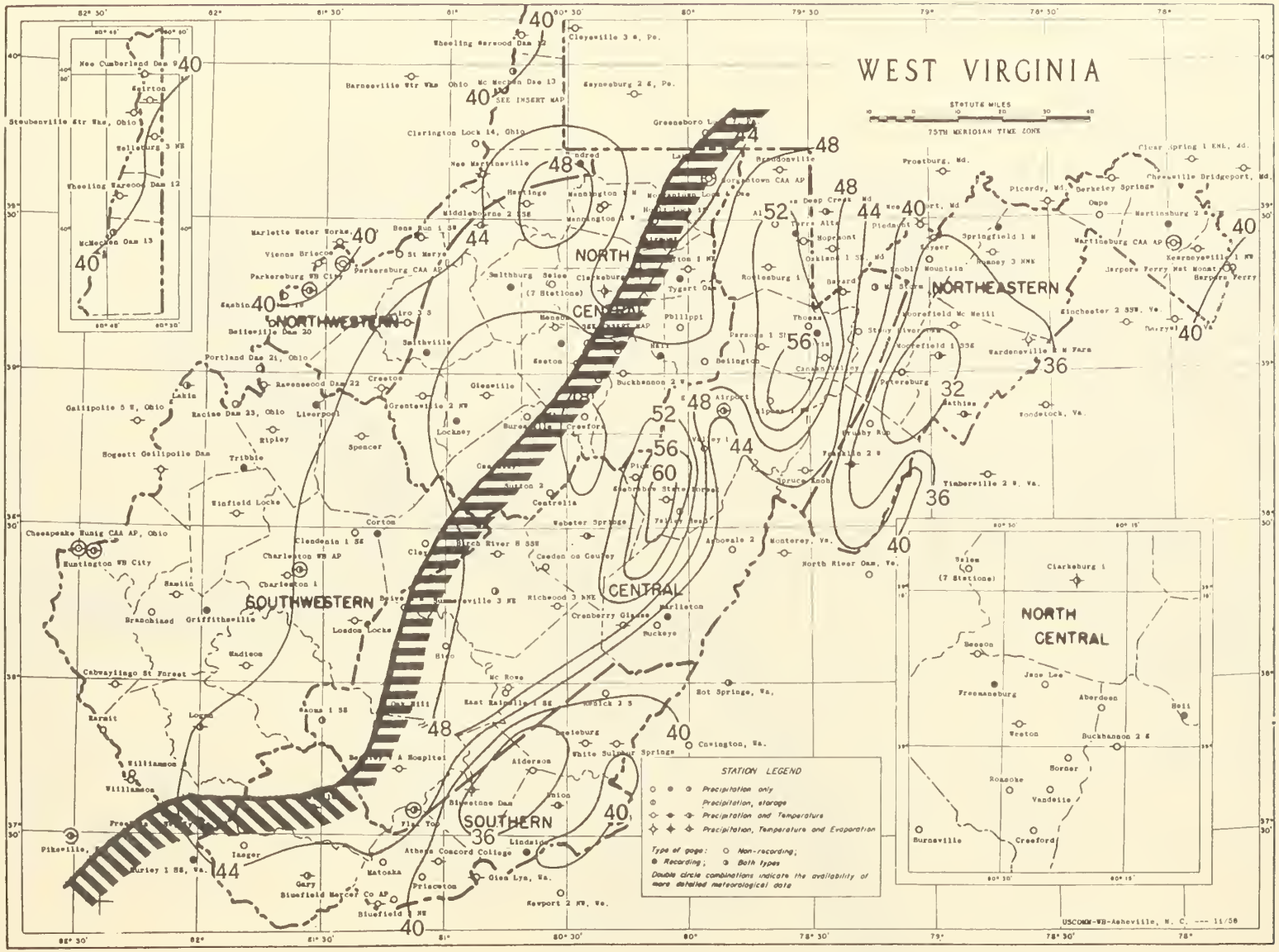
Mean Annual Precipitation, Inches



Based on period 1931-55

Figure 2. Isolines are drawn through points of approximately equal value. Caution should be used in interpolating on these maps, particularly in mountainous areas.

Mean Annual Precipitation, Inches



Based on period 1931-55

Figure 3. Isolines are drawn through points of approximately equal value. Caution should be used in interpolating on these maps, particularly in mountainous areas.

Mean Annual Precipitation, Inches

Based on period 1931-55

Figure 4. Isolines are drawn through points of approximately equal value. Caution should be used in interpolating on these maps, particularly in mountainous areas.

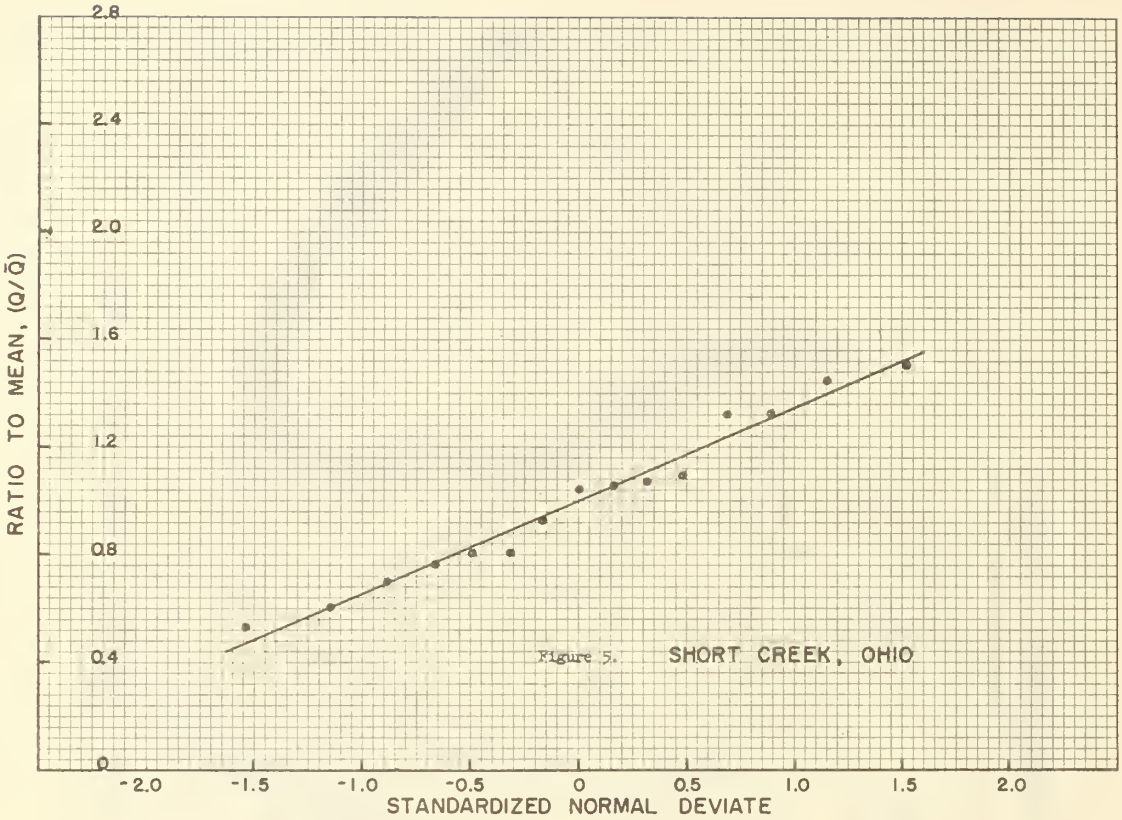


Figure 5. SHORT CREEK, OHIO

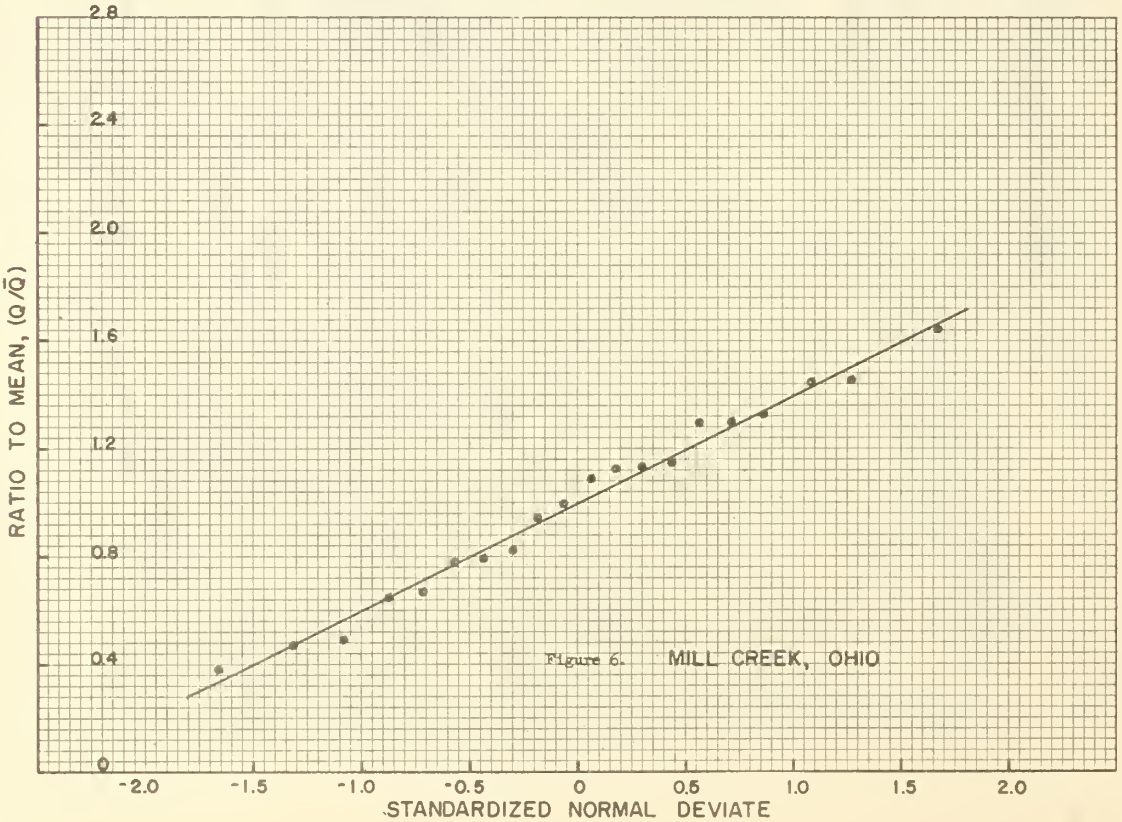


Figure 6. MILL CREEK, OHIO

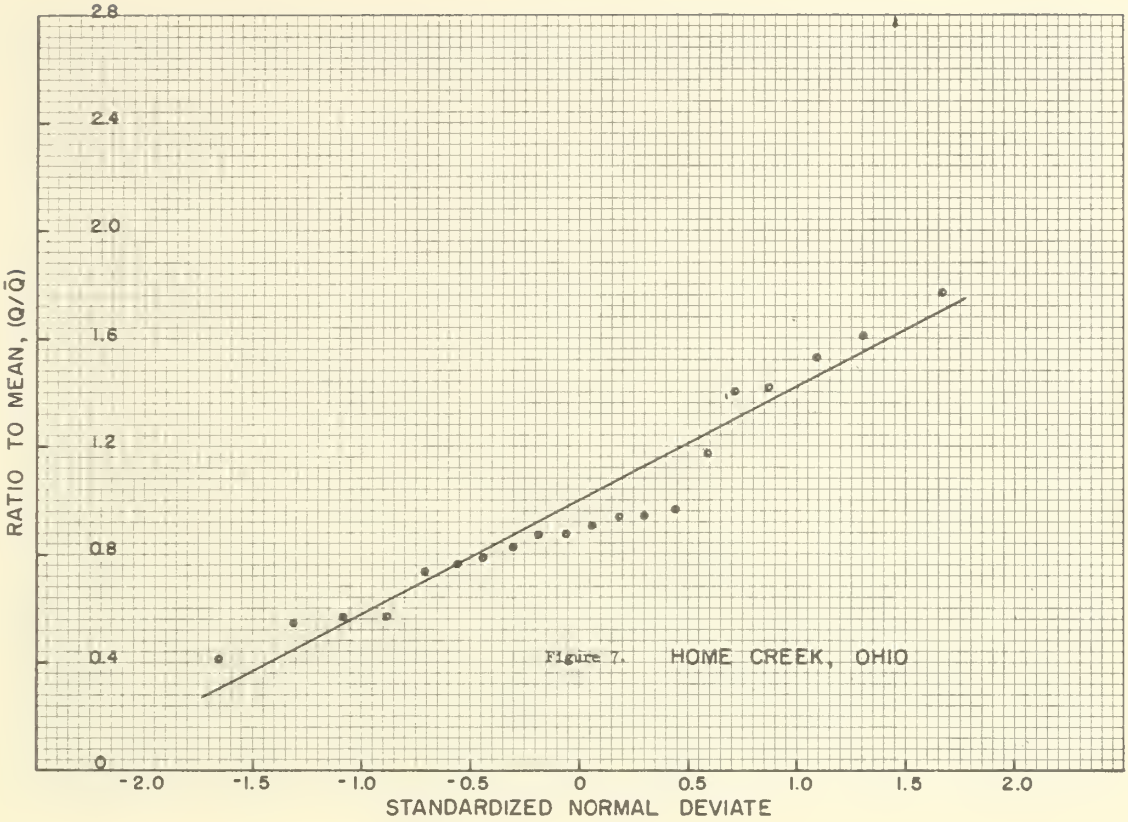


Figure 7. HOME CREEK, OHIO

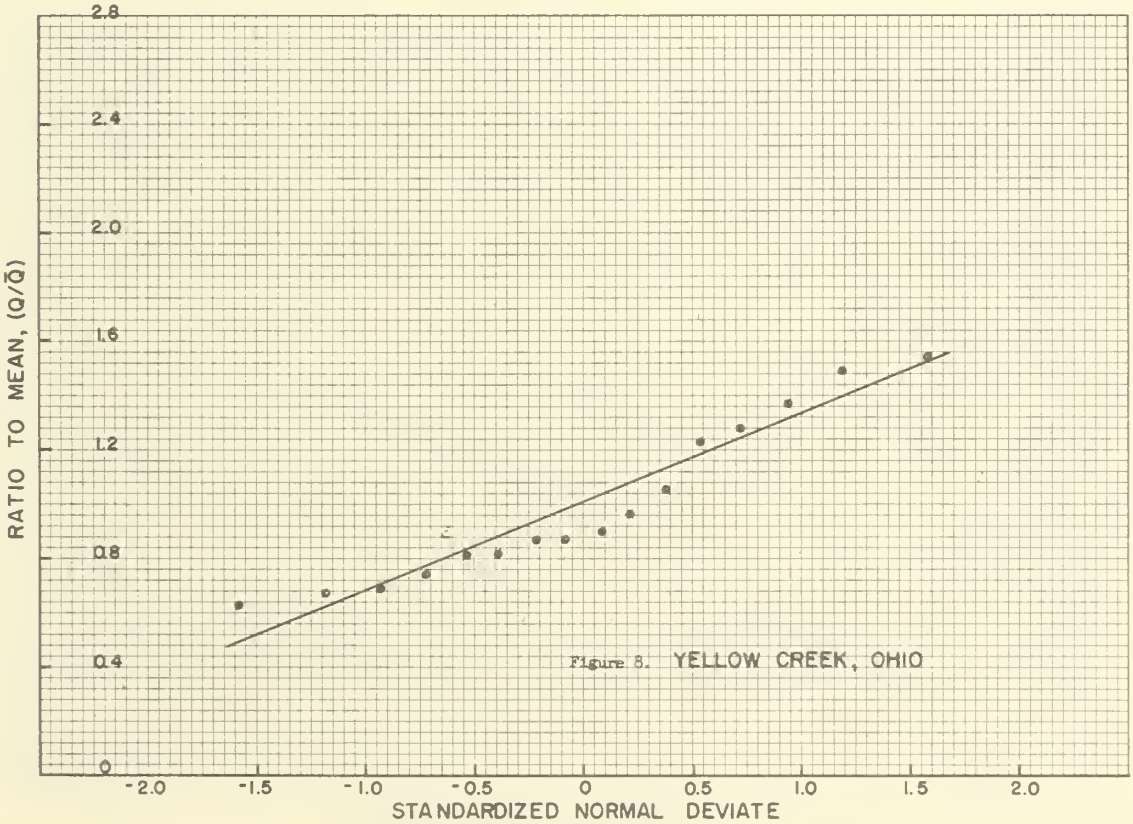


Figure 8. YELLOW CREEK, OHIO

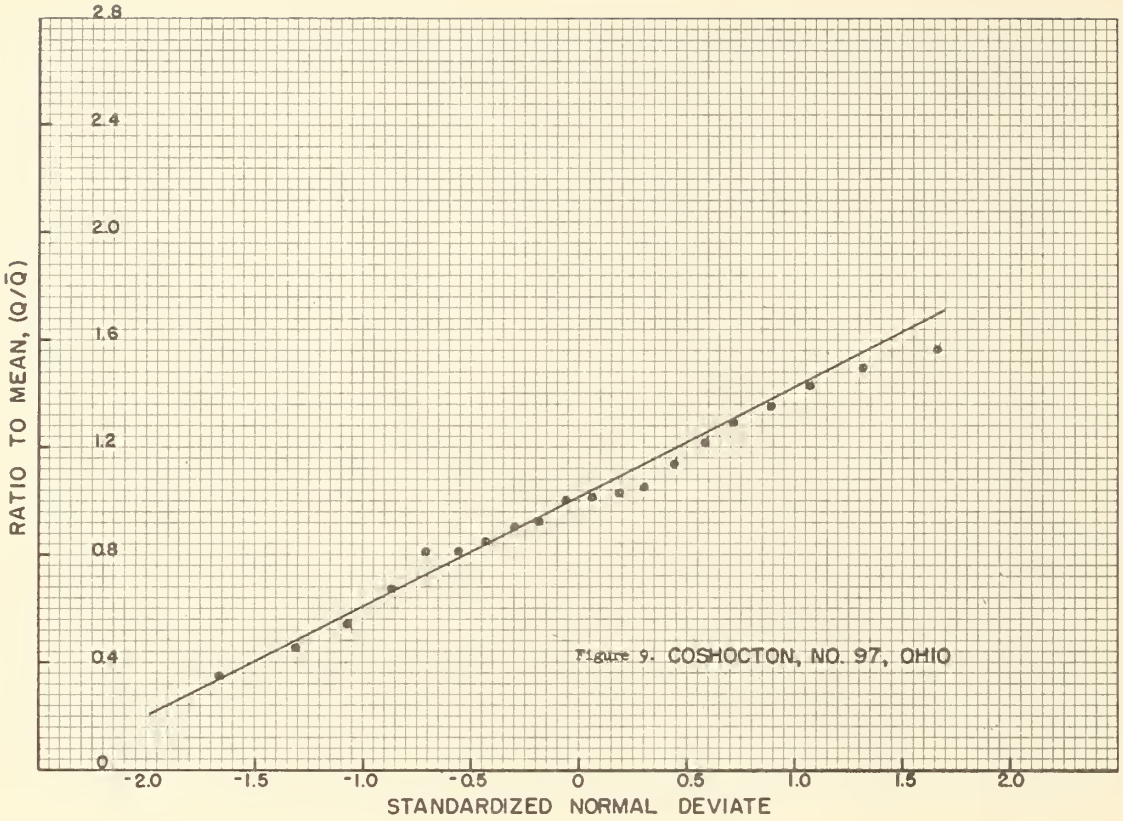


Figure 9. COSHOCTON, NO. 97, OHIO

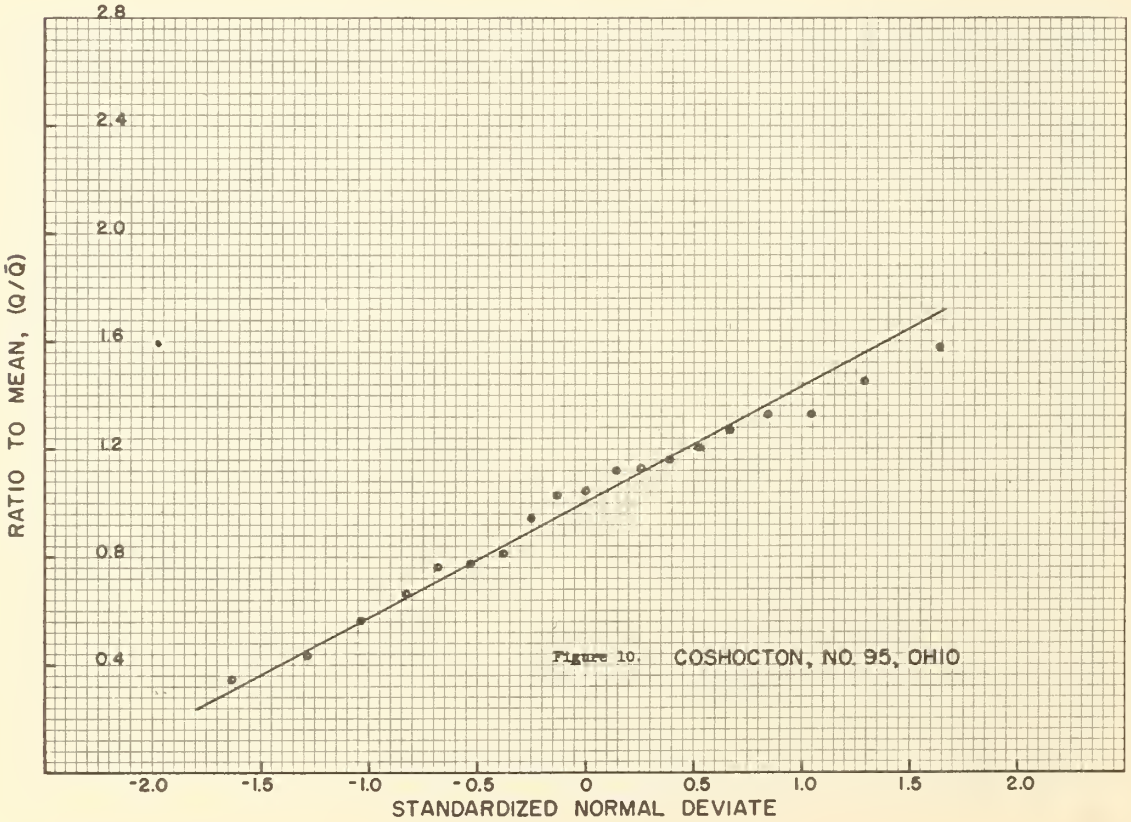


Figure 10. COSHOCTON, NO. 95, OHIO

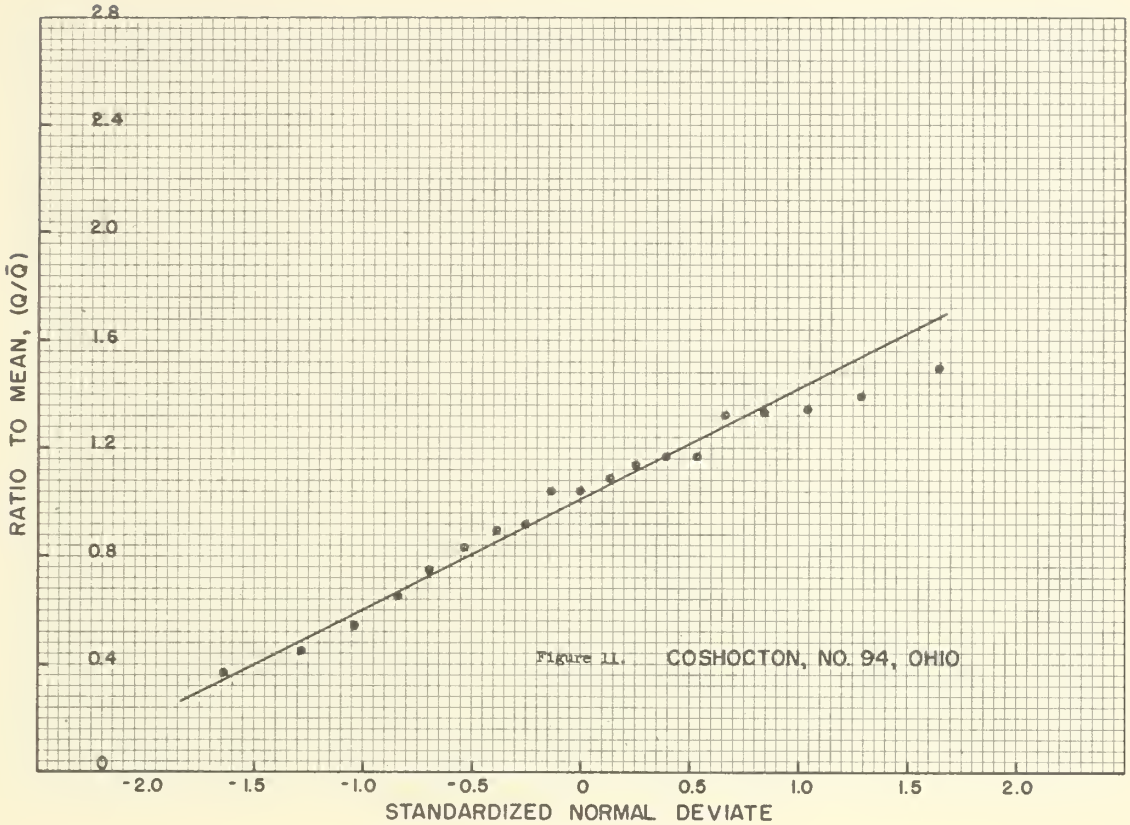


Figure 11. COSHOCTON, NO. 94, OHIO

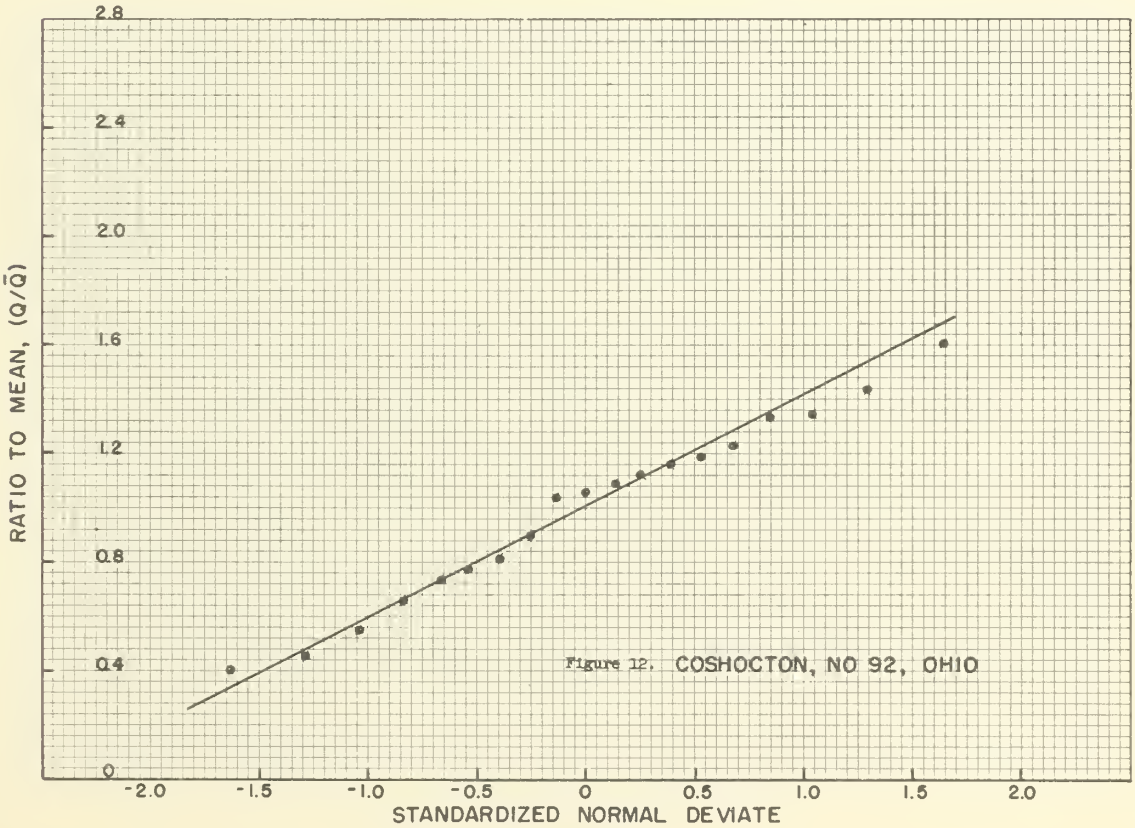


Figure 12. COSHOCTON, NO 92, OHIO

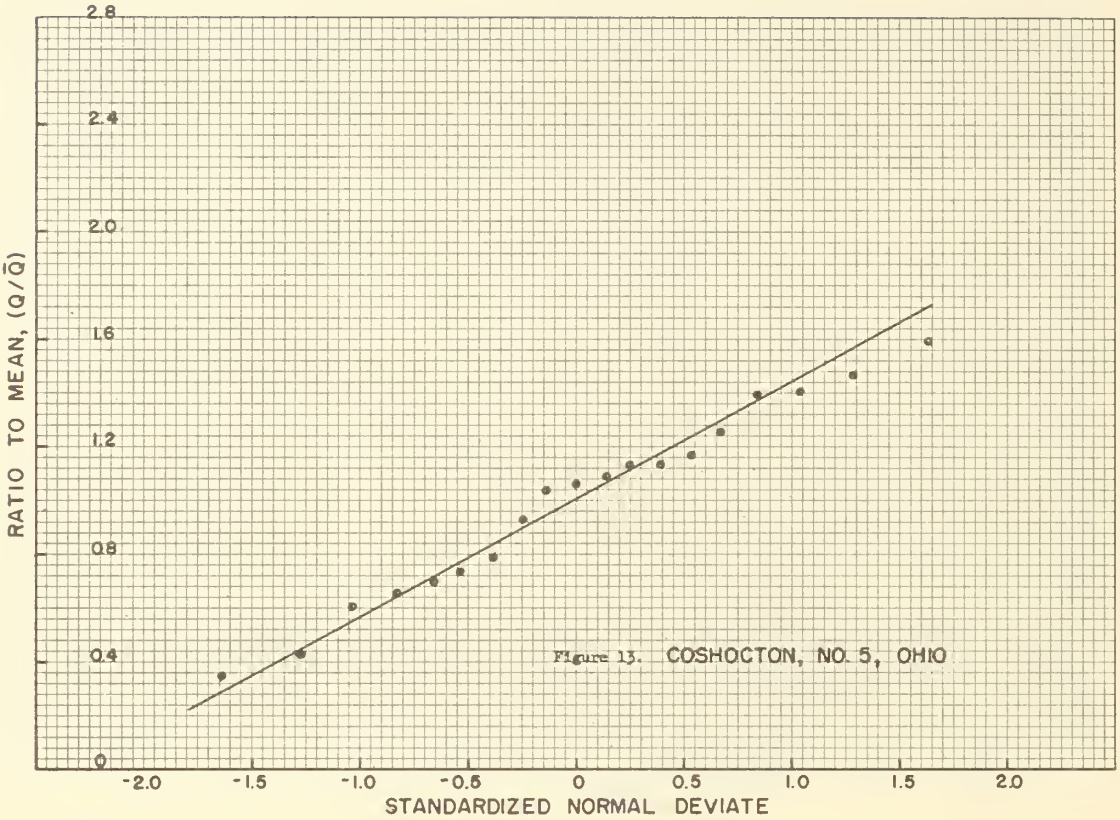


Figure 13. COSHOCTON, NO. 5, OHIO

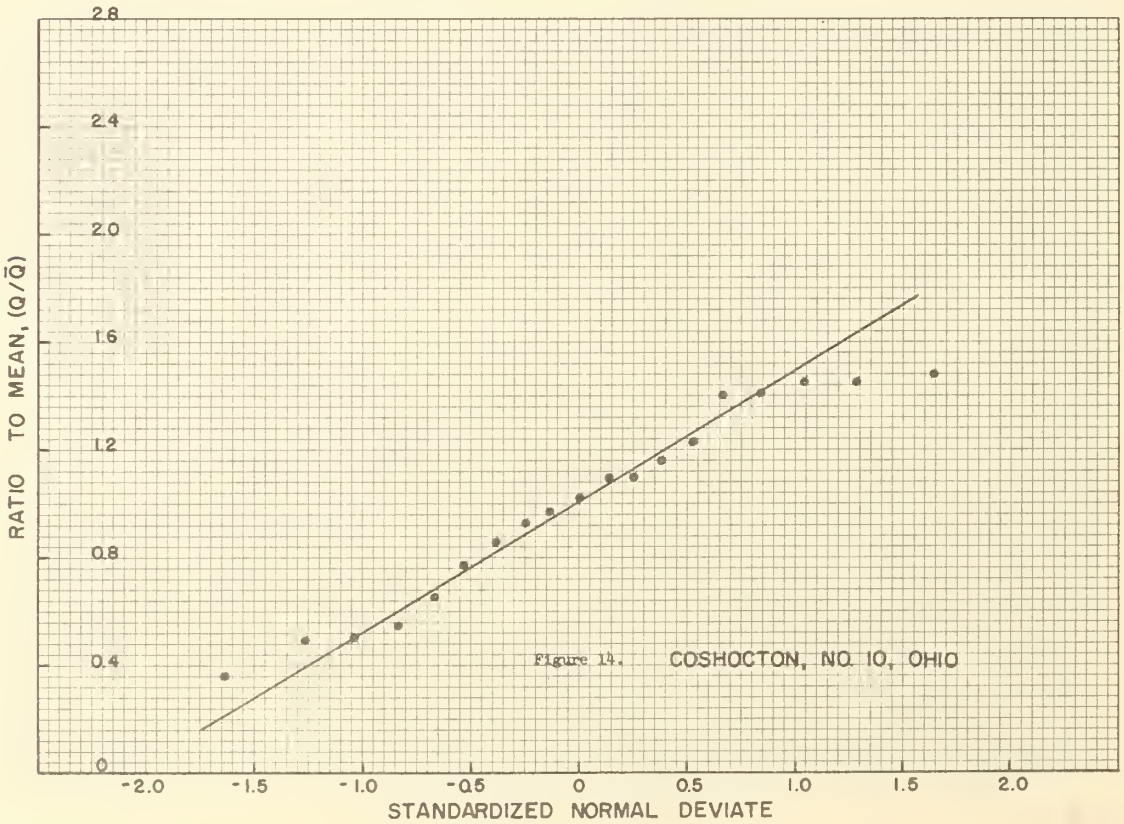


Figure 14. COSHOCTON, NO. 10, OHIO

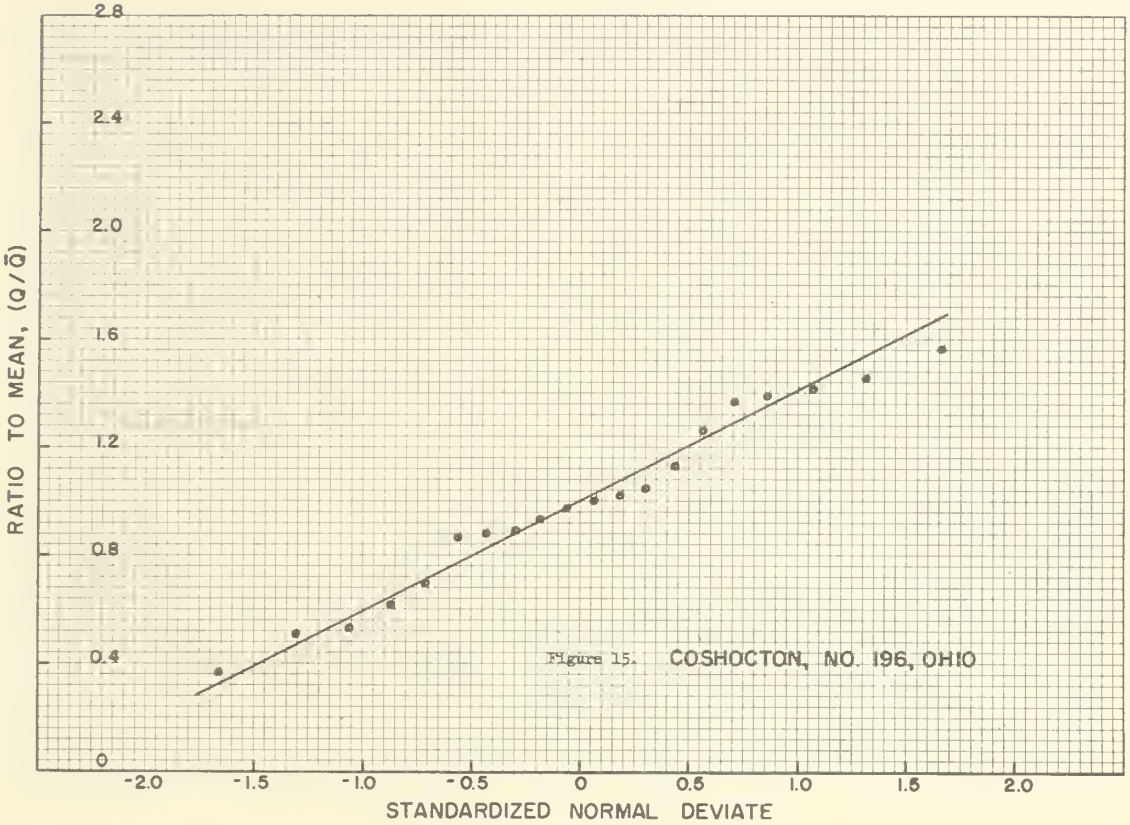


Figure 15. COSHOCTON, NO. 196, OHIO

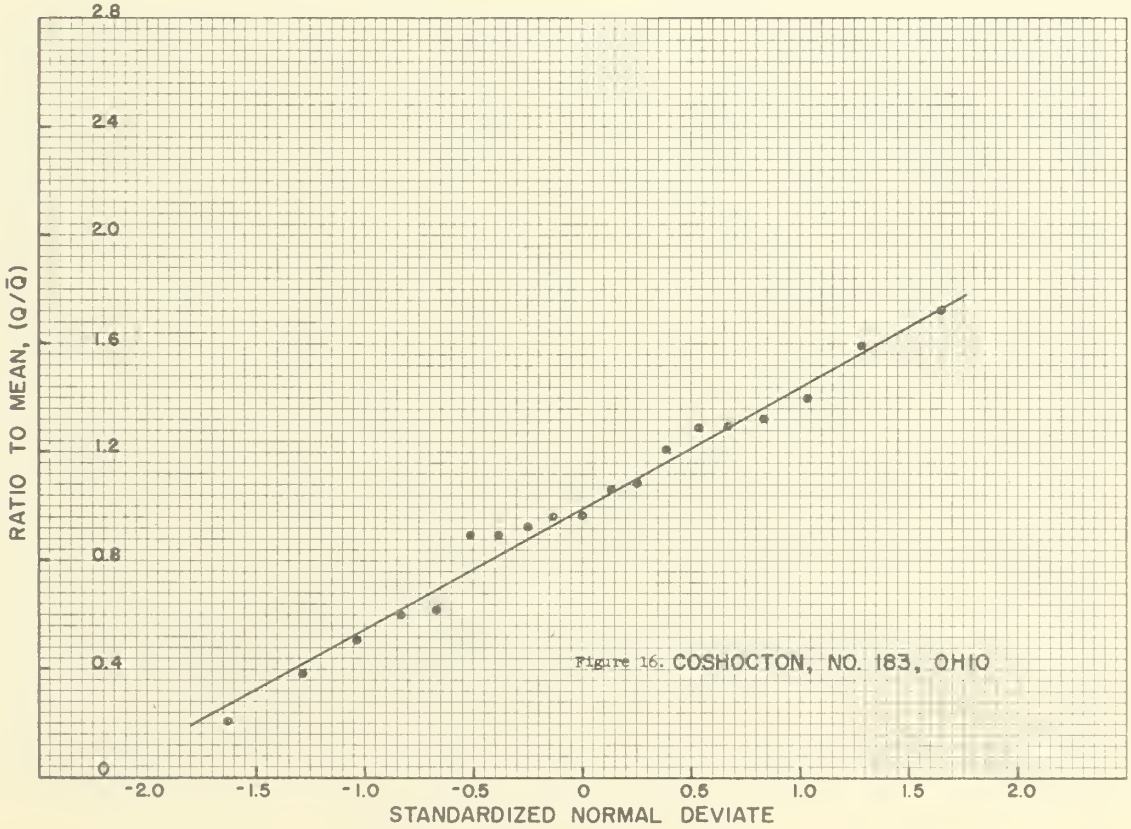


Figure 16. COSHOCTON, NO. 183, OHIO

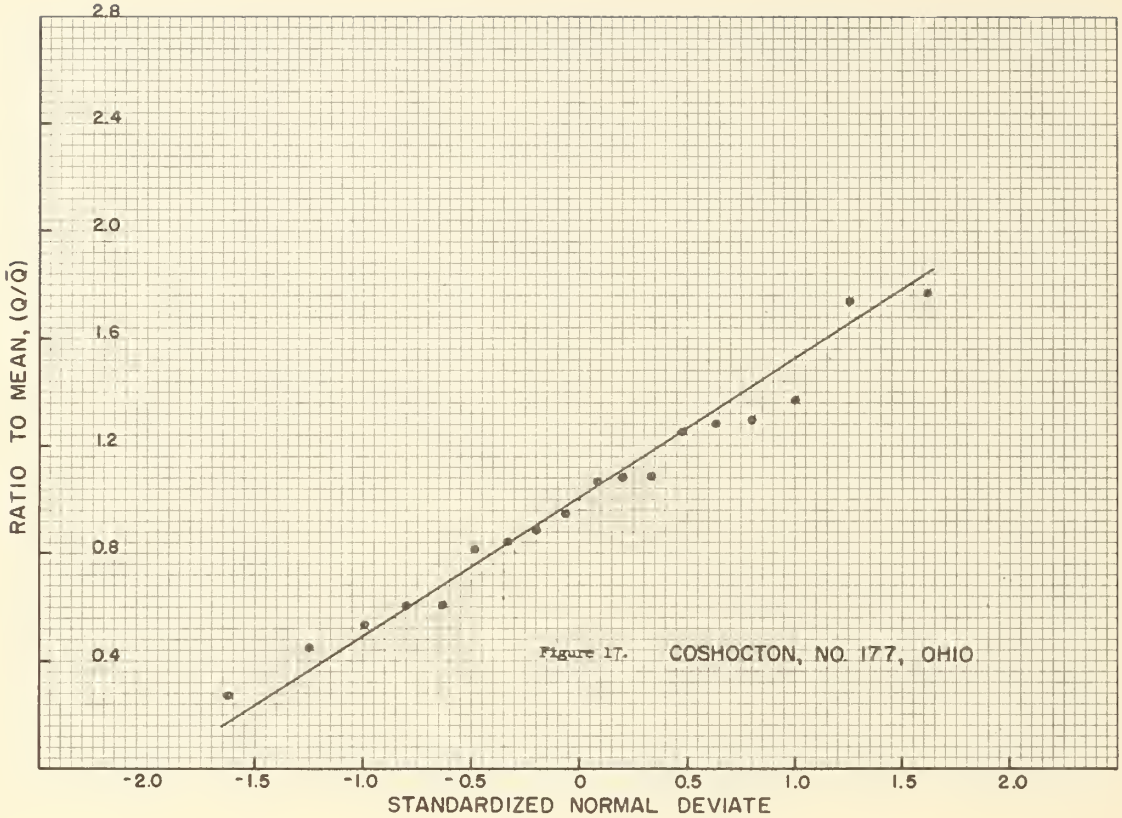


Figure 17. COSHOCTON, NO. 177, OHIO

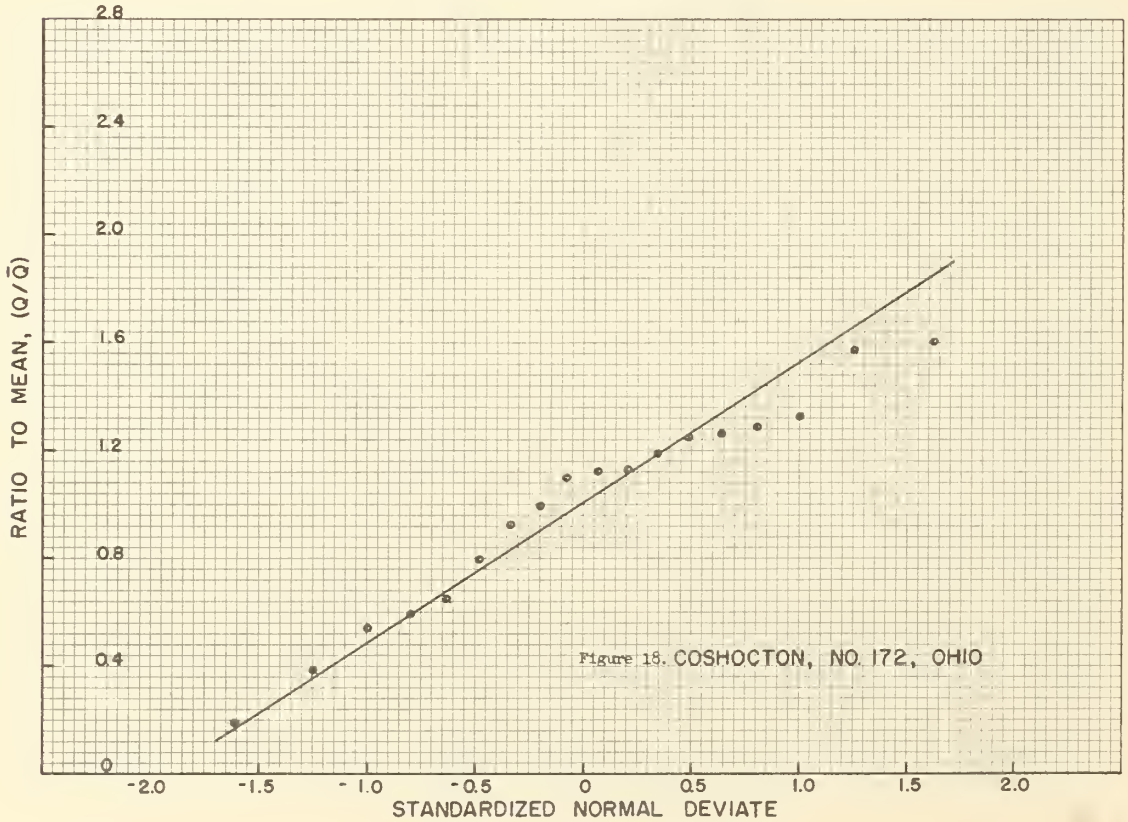


Figure 18. COSHOCTON, NO. 172, OHIO

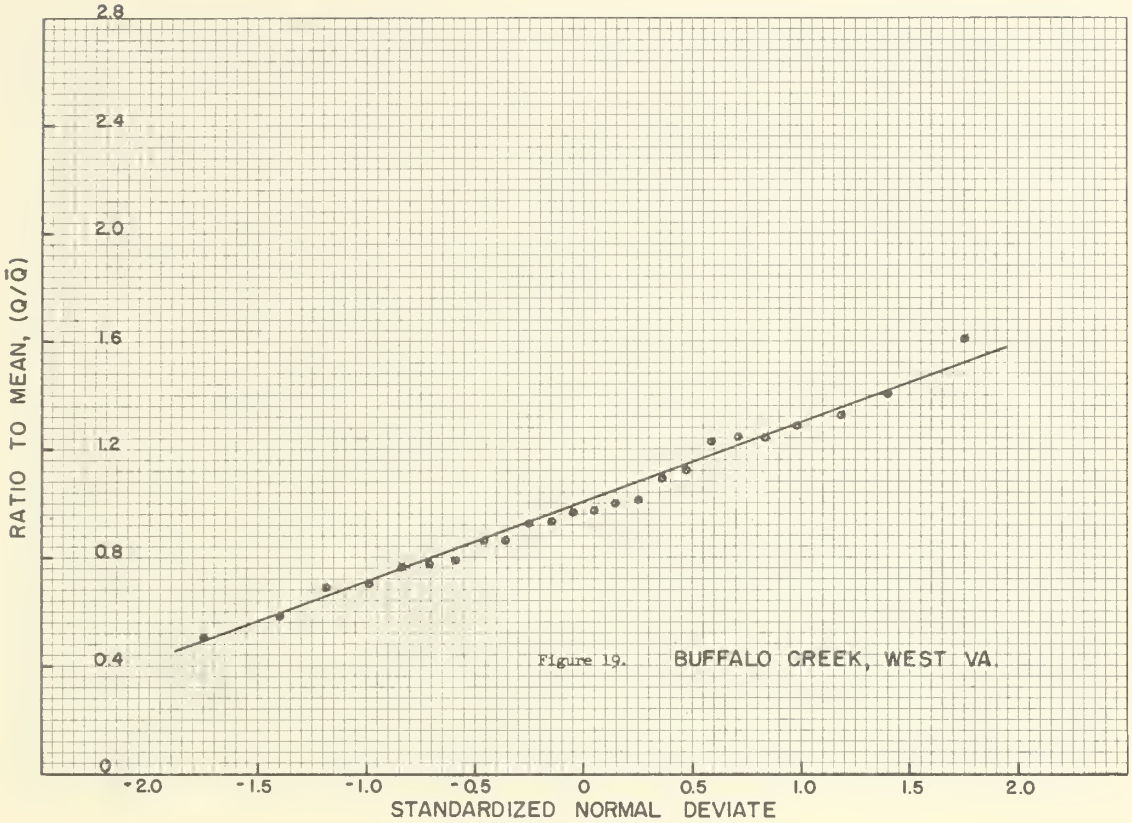


Figure 19. BUFFALO CREEK, WEST VA.

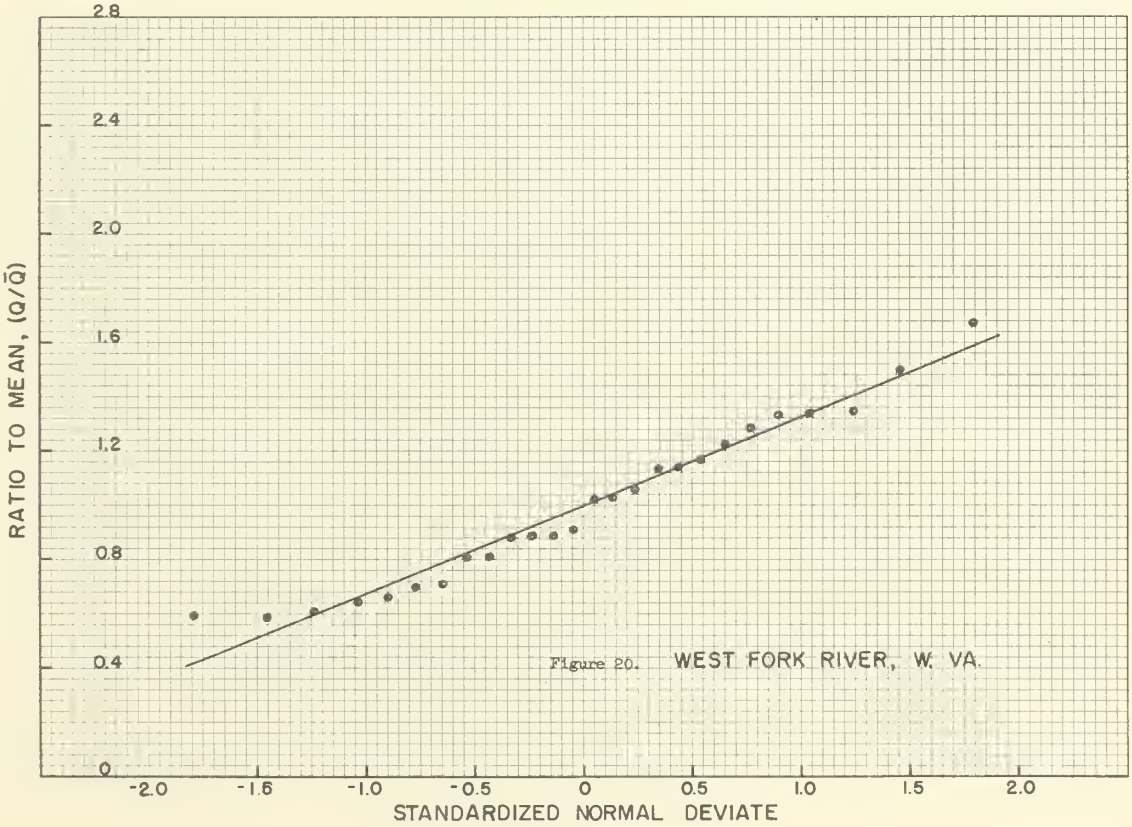


Figure 20. WEST FORK RIVER, W. VA.

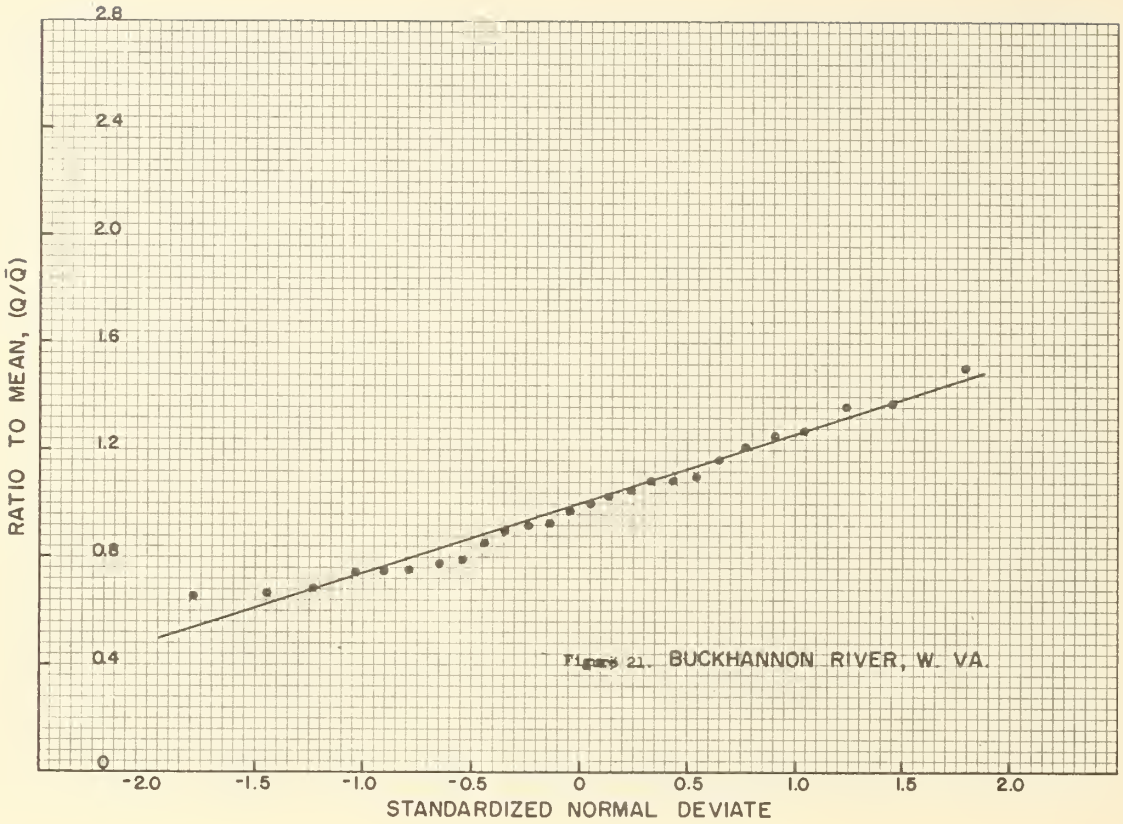


Figure 21. BUCKHANNON RIVER, W. VA.

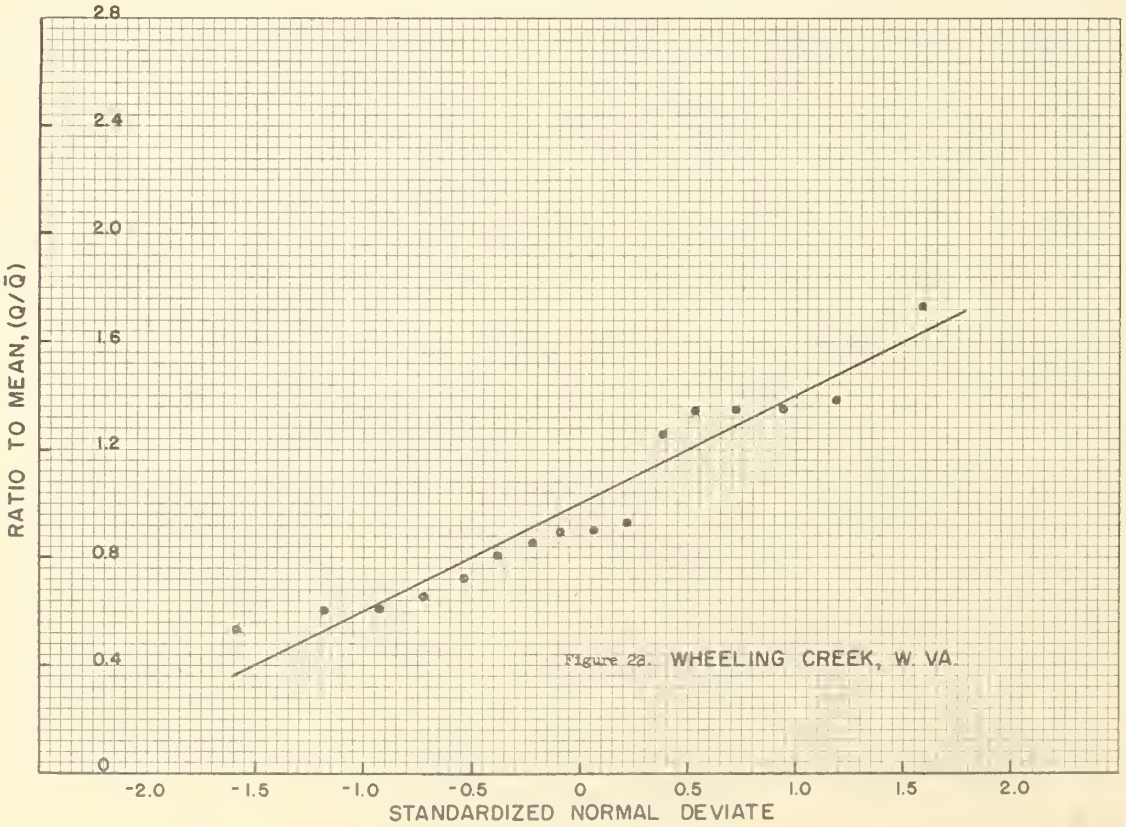


Figure 22. WHEELING CREEK, W. VA.

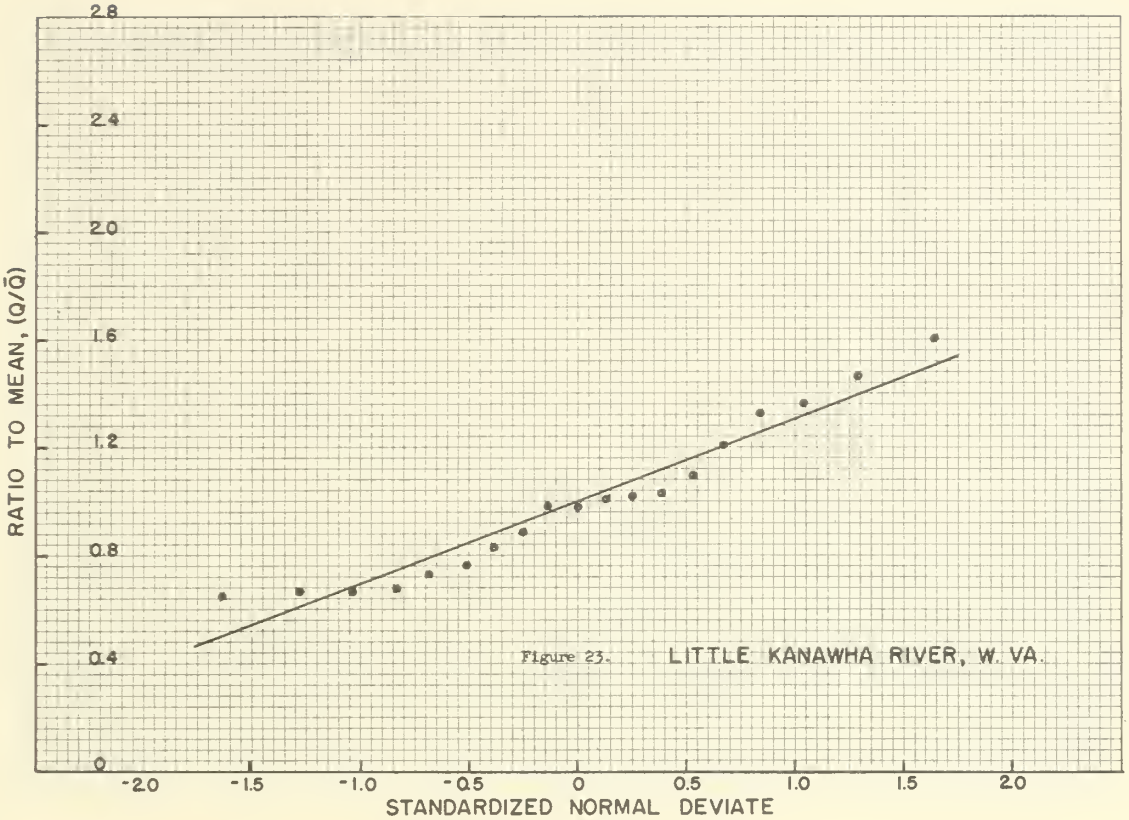


Figure 23. LITTLE KANAWHA RIVER, W. VA.

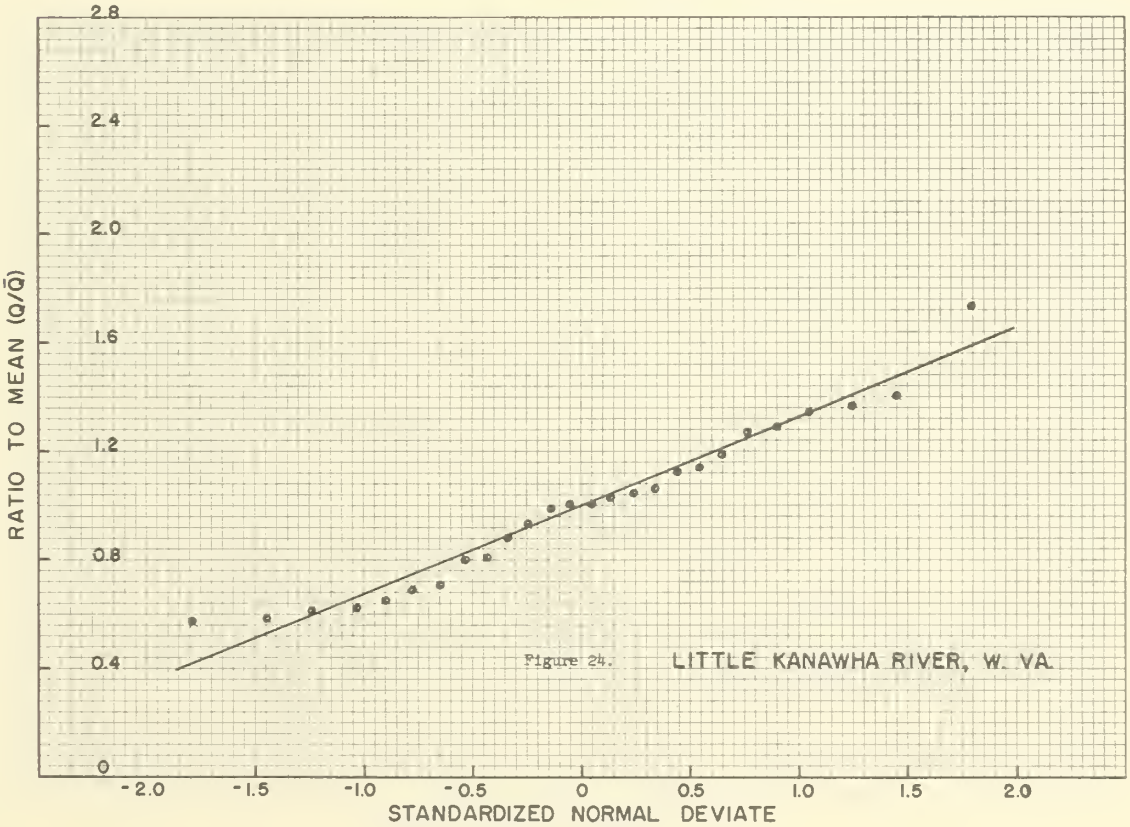


Figure 24. LITTLE KANAWHA RIVER, W. VA.

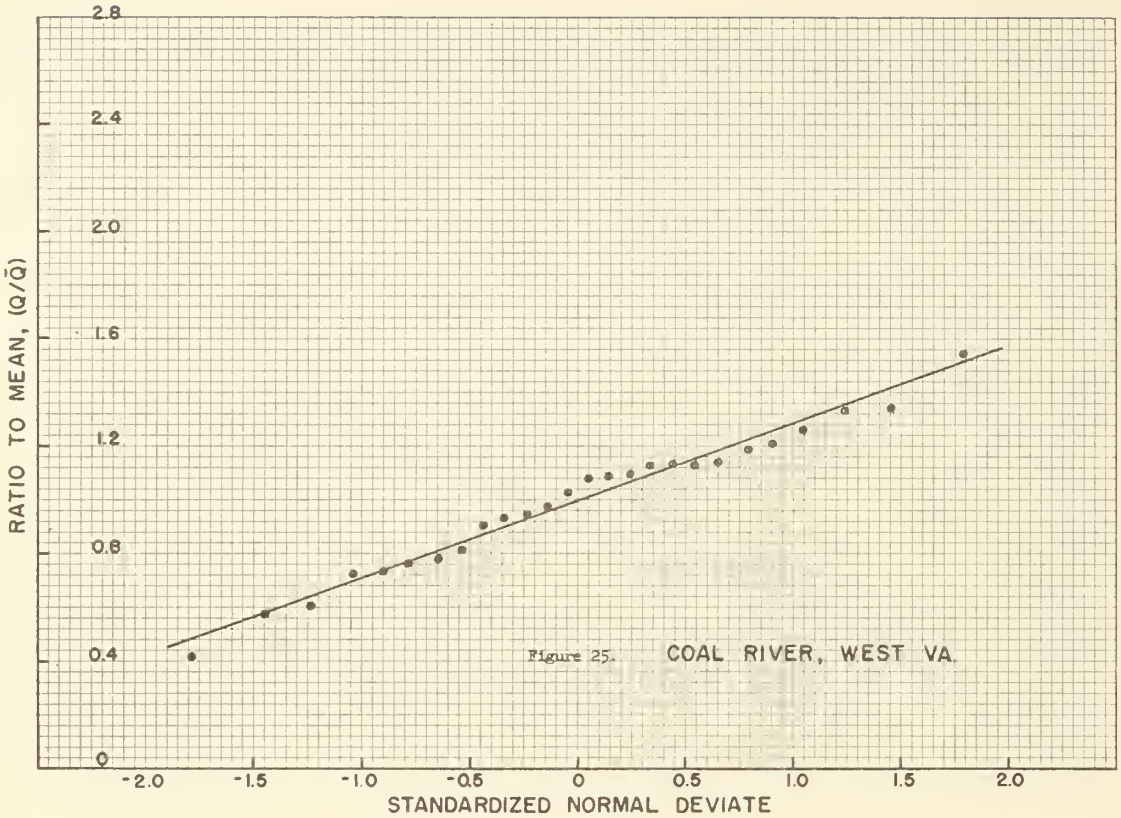


Figure 25. COAL RIVER, WEST VA.

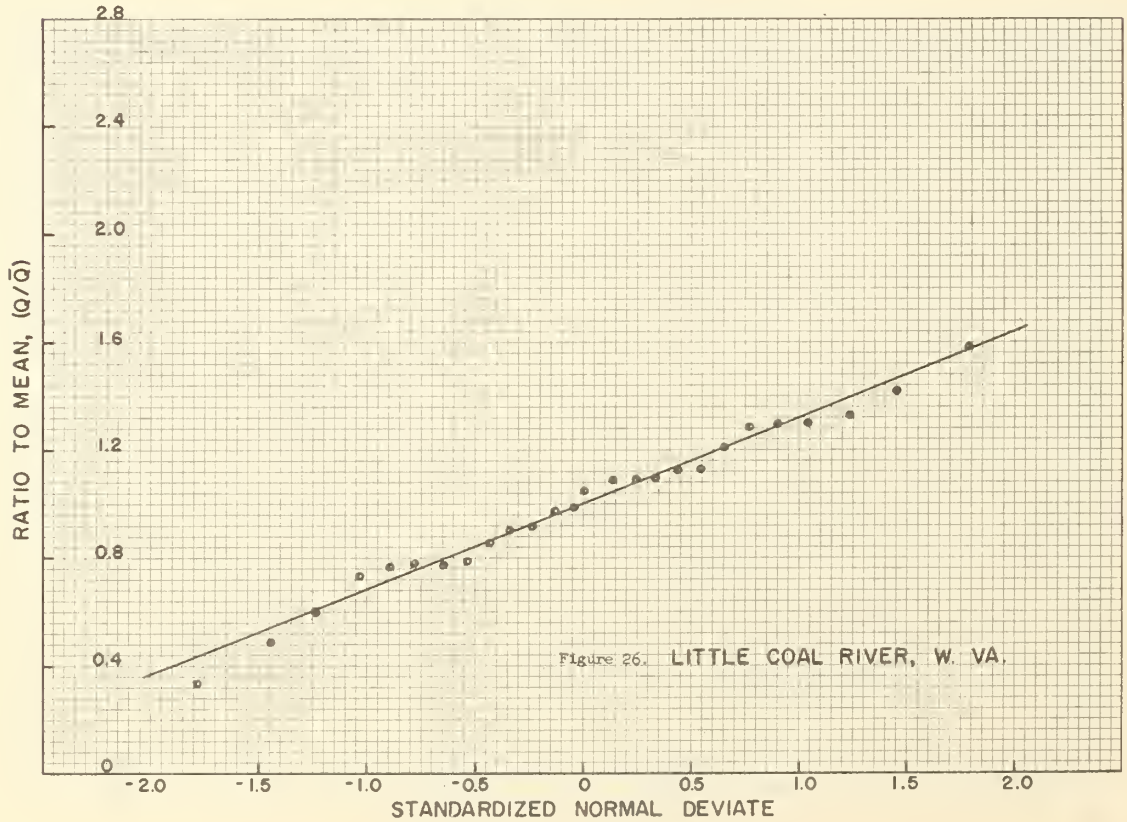


Figure 26. LITTLE COAL RIVER, W. VA.

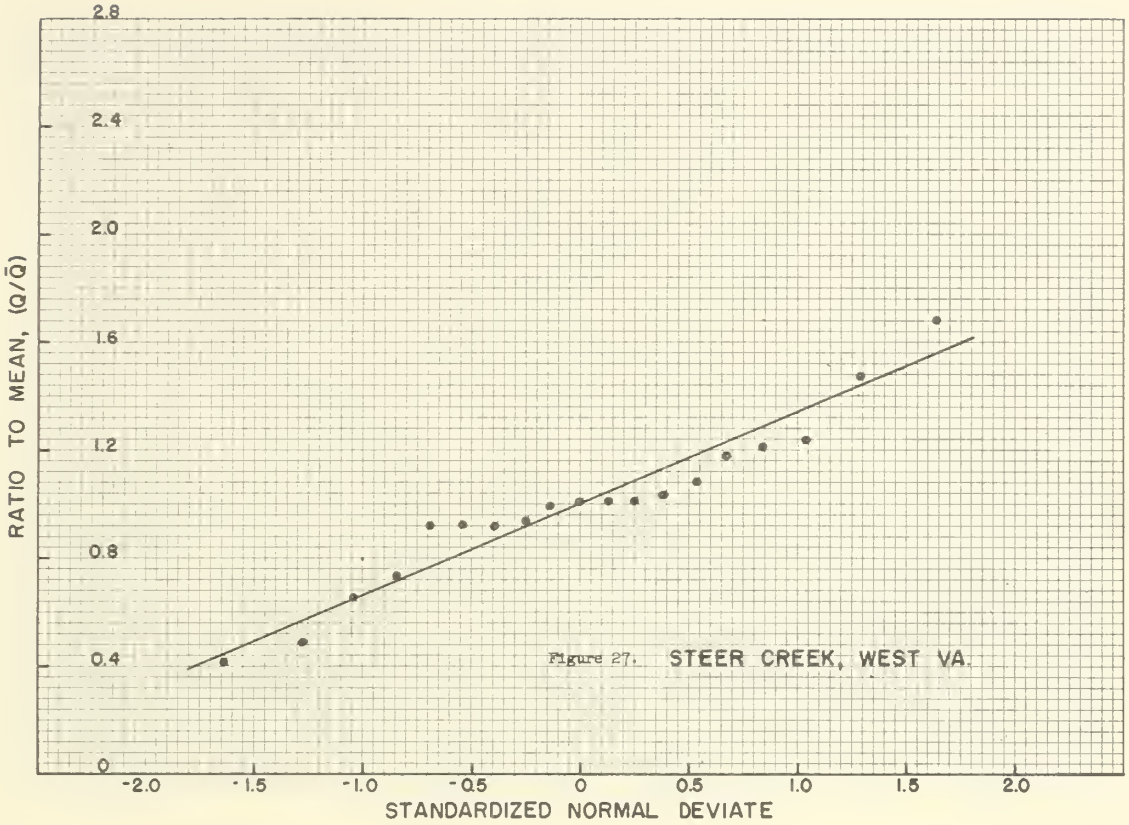


Figure 27. STEER CREEK, WEST VA.

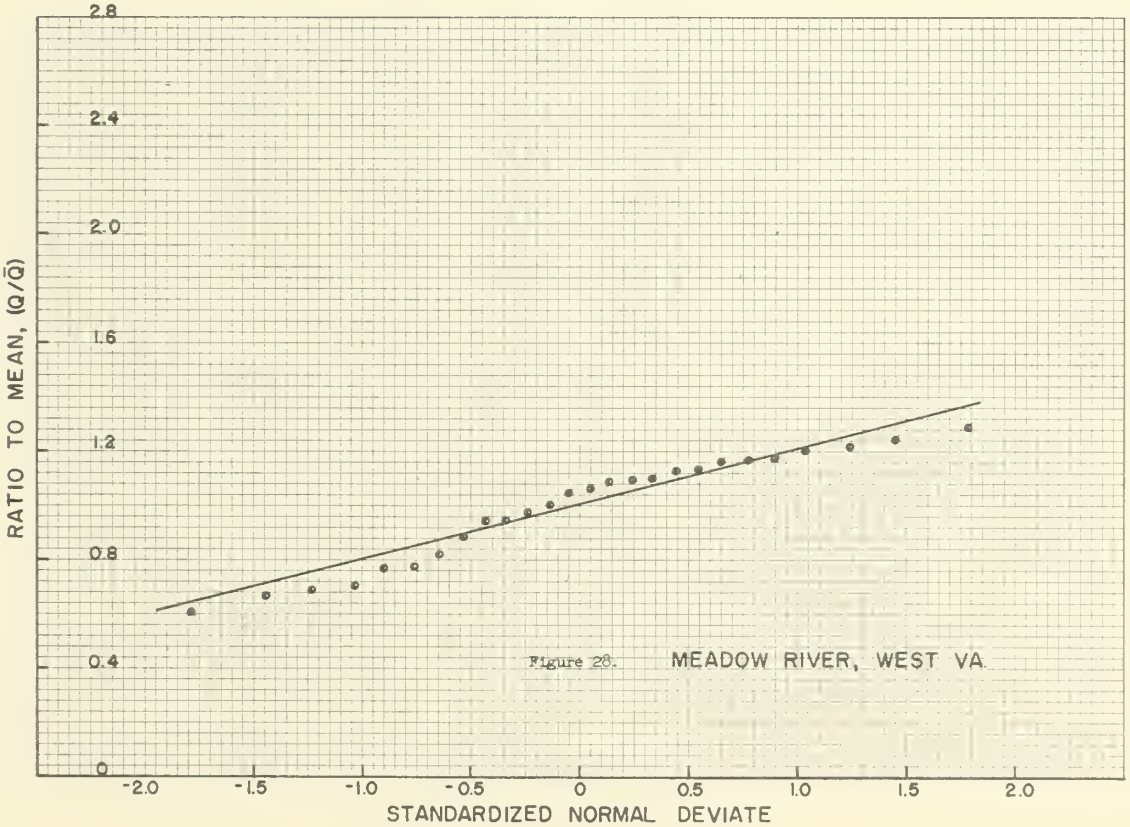


Figure 28. MEADOW RIVER, WEST VA.

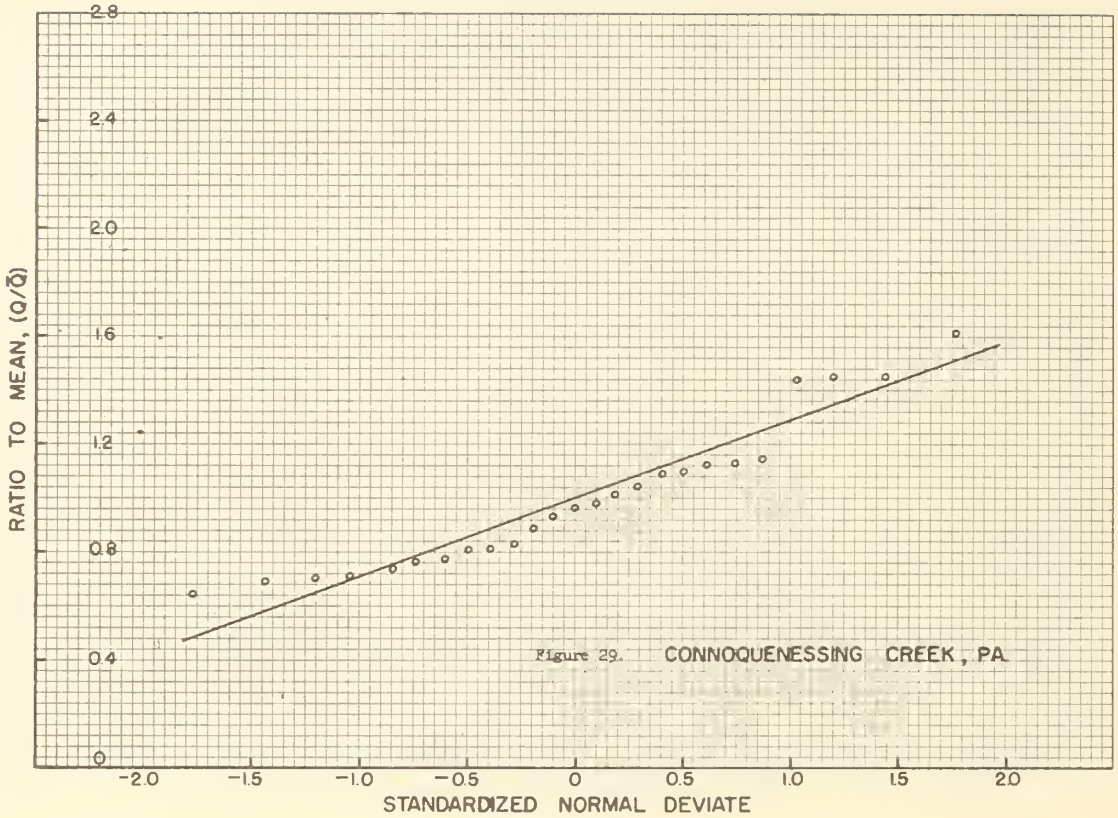


Figure 29. CONNOQUENESSING CREEK, PA.

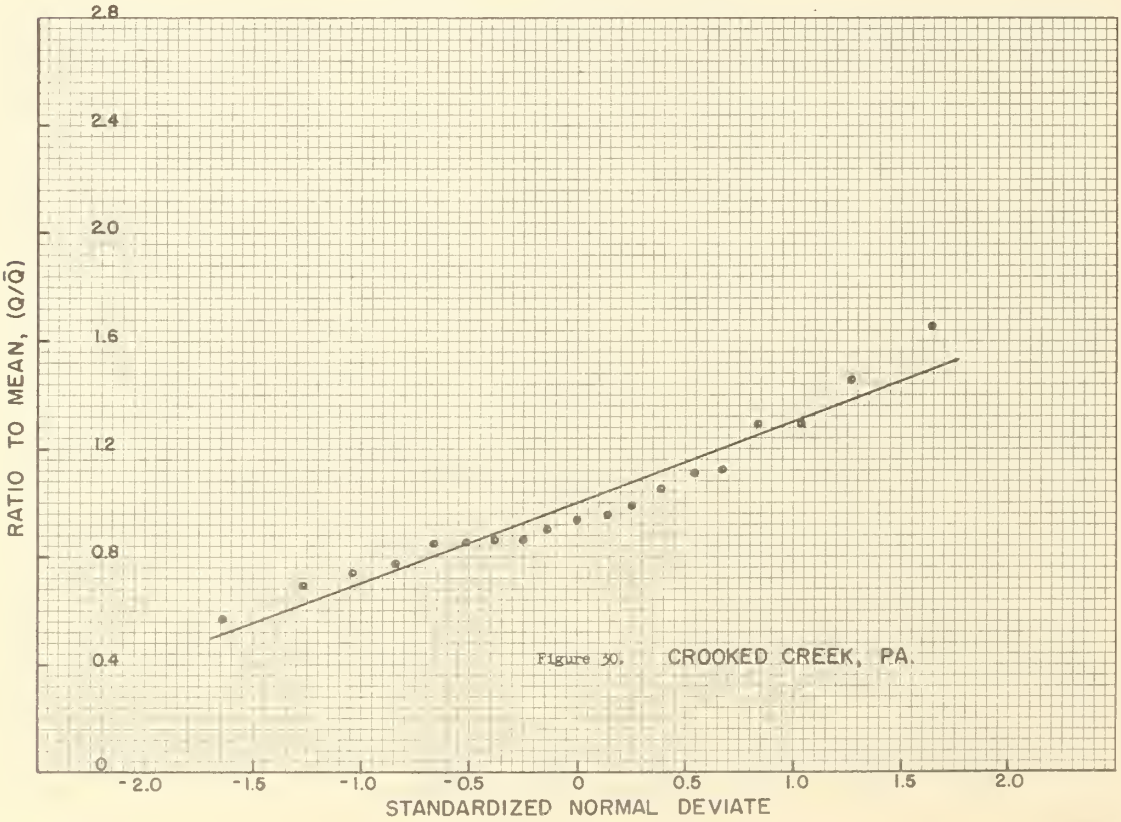


Figure 30. CROOKED CREEK, PA.

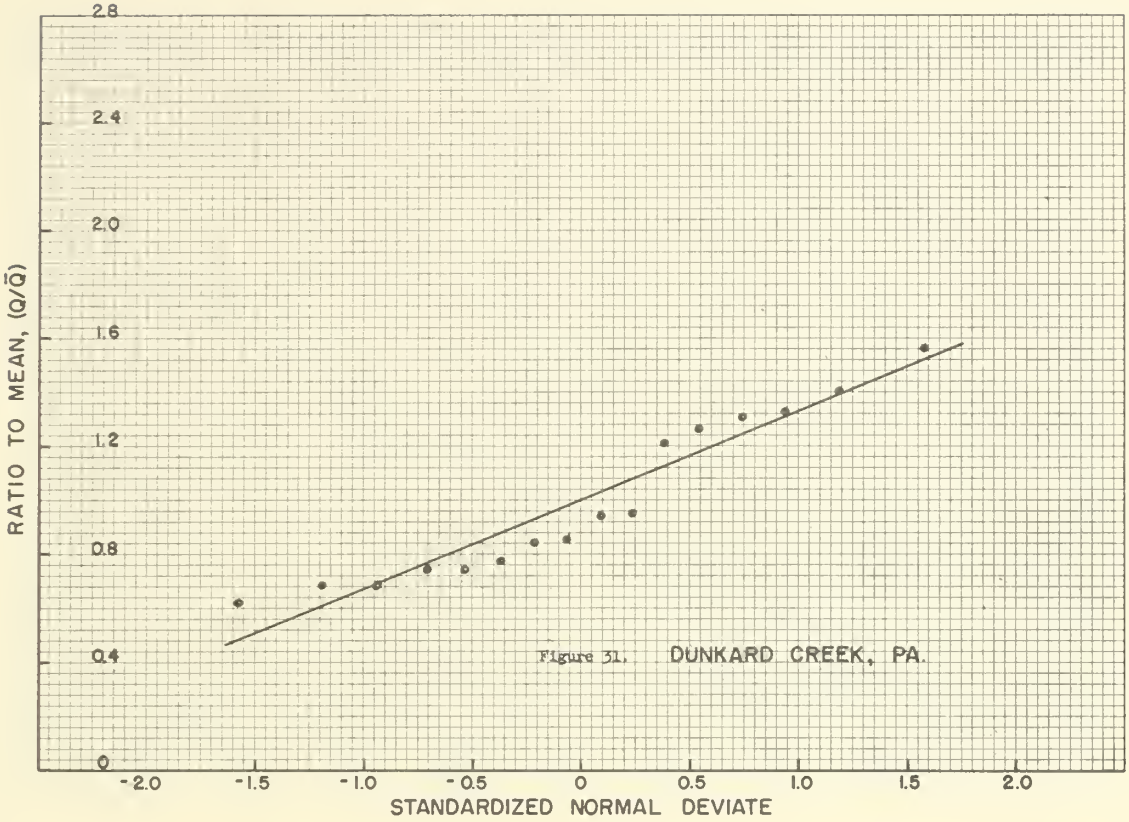


Figure 31. DUNKARD CREEK, PA.

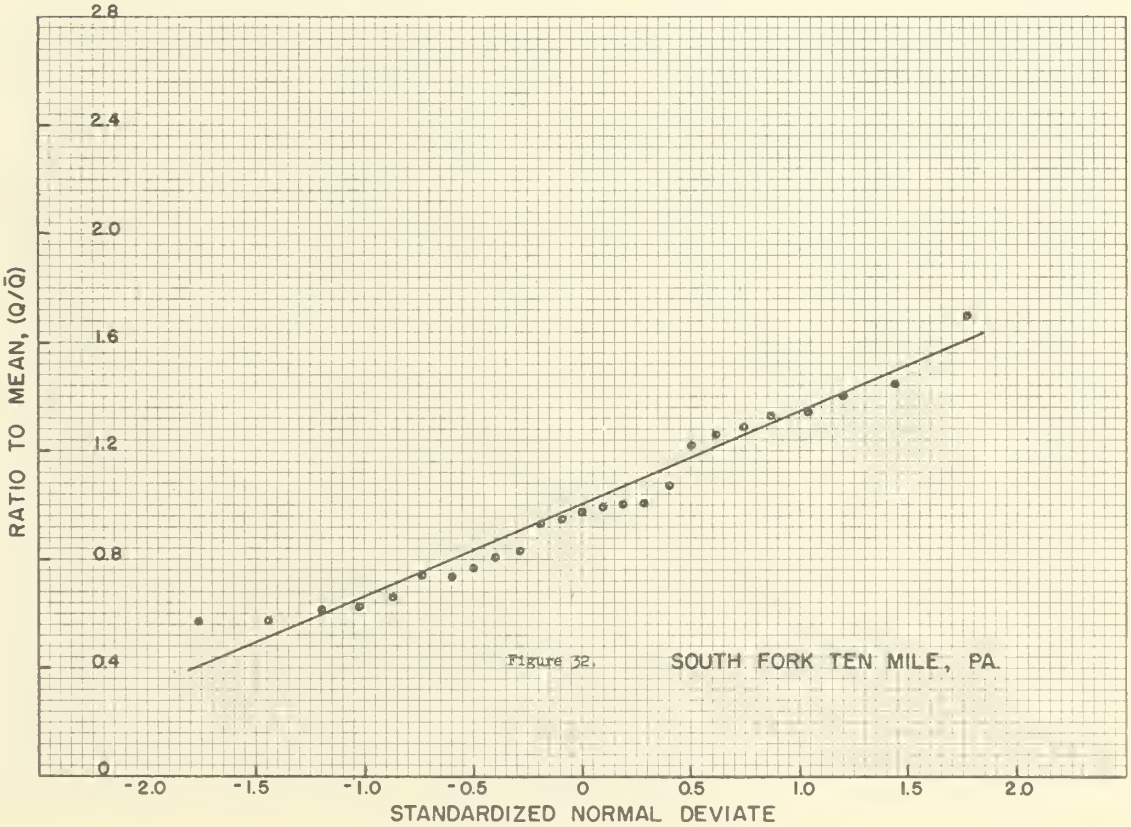


Figure 32. SOUTH FORK TEN MILE, PA.

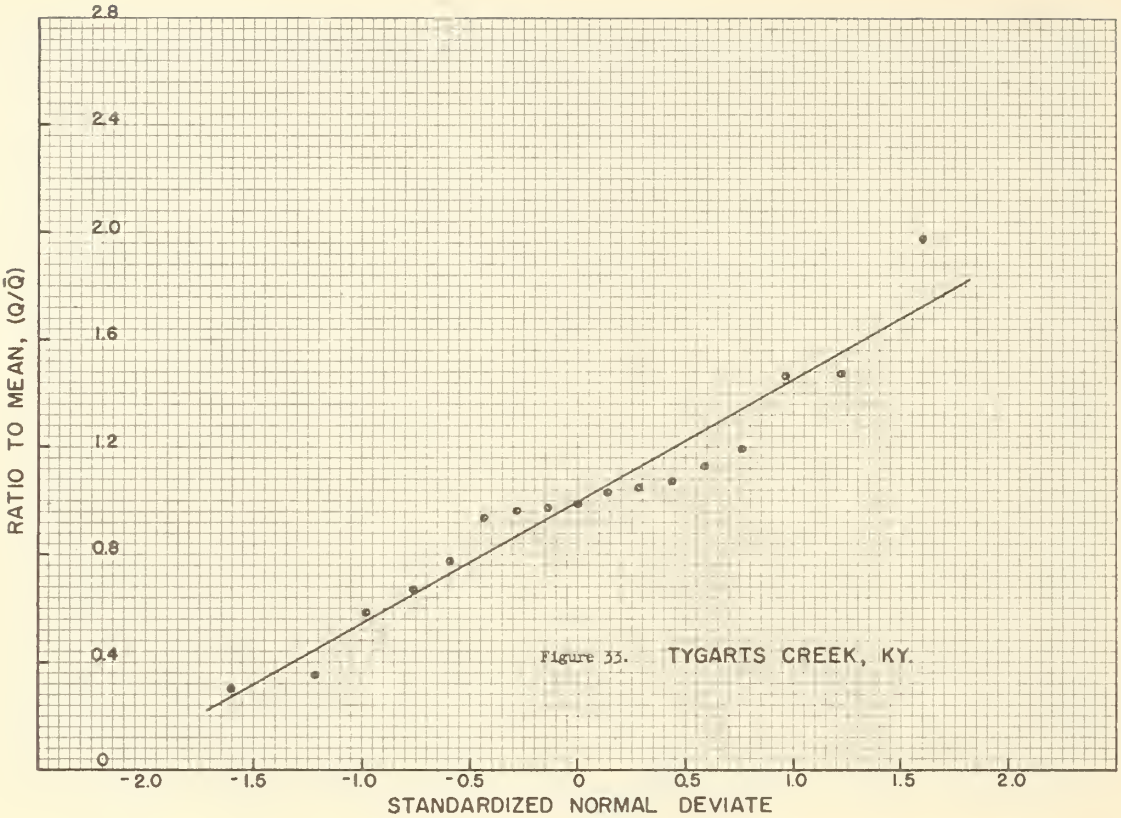


Figure 33. TYGARTS CREEK, KY.

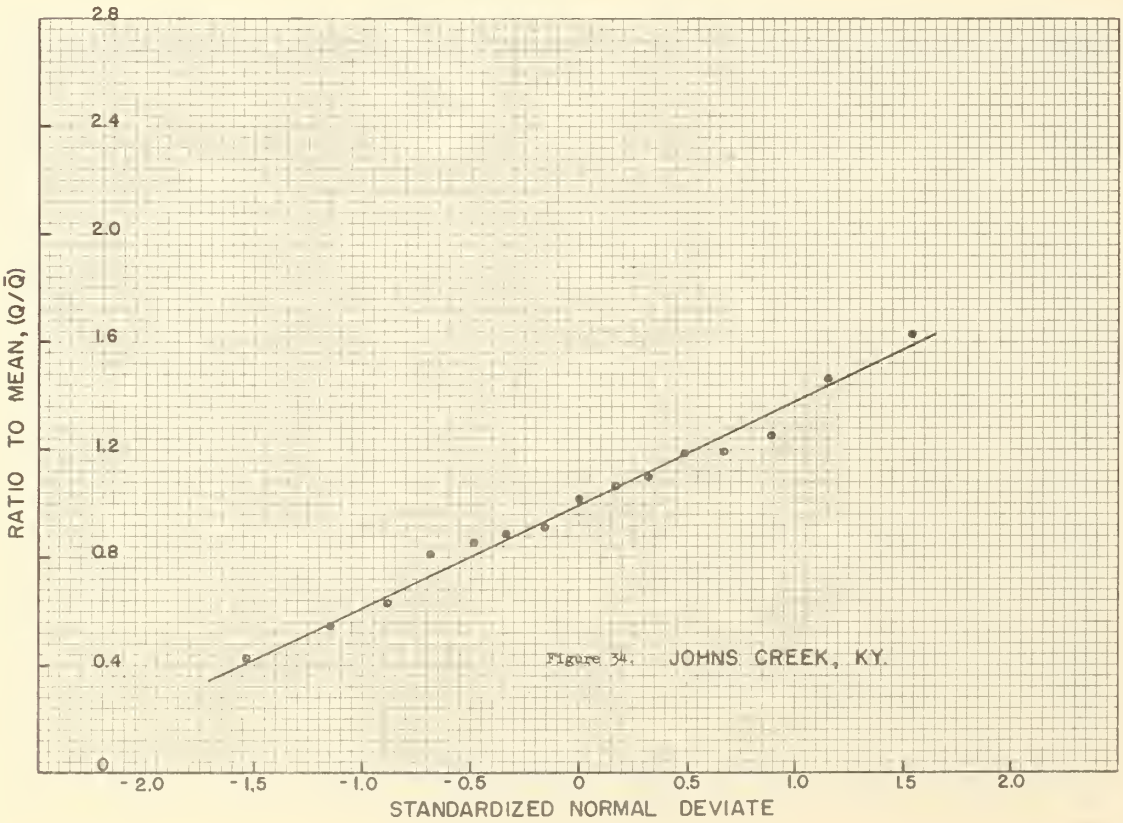


Figure 34. JOHNS CREEK, KY.

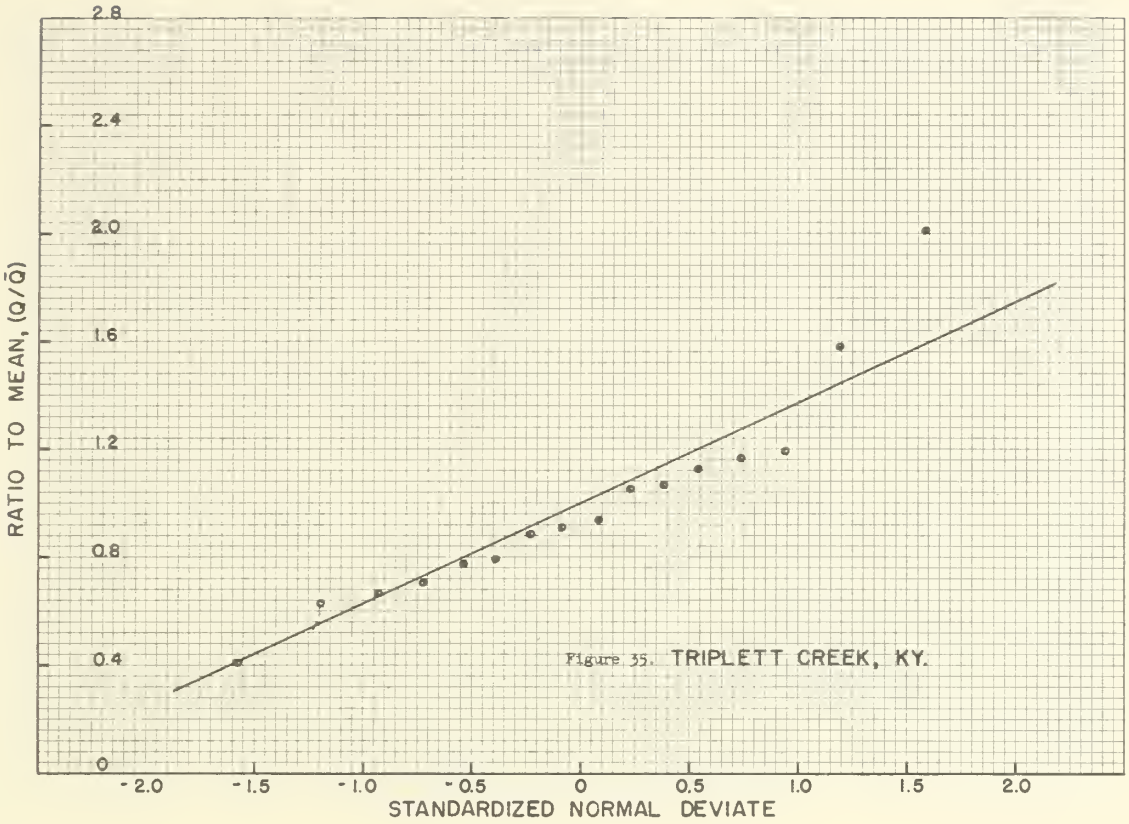


Figure 35. TRIPLETT CREEK, KY.

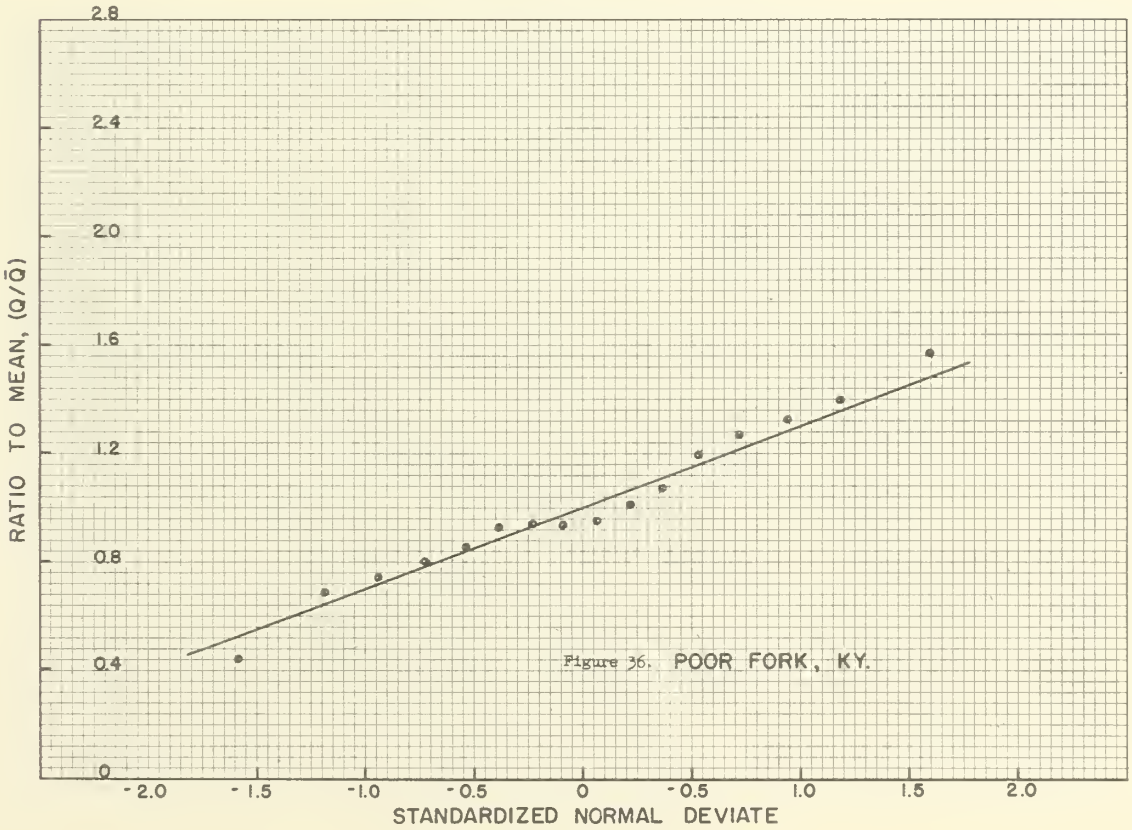


Figure 36. POOR FORK, KY.

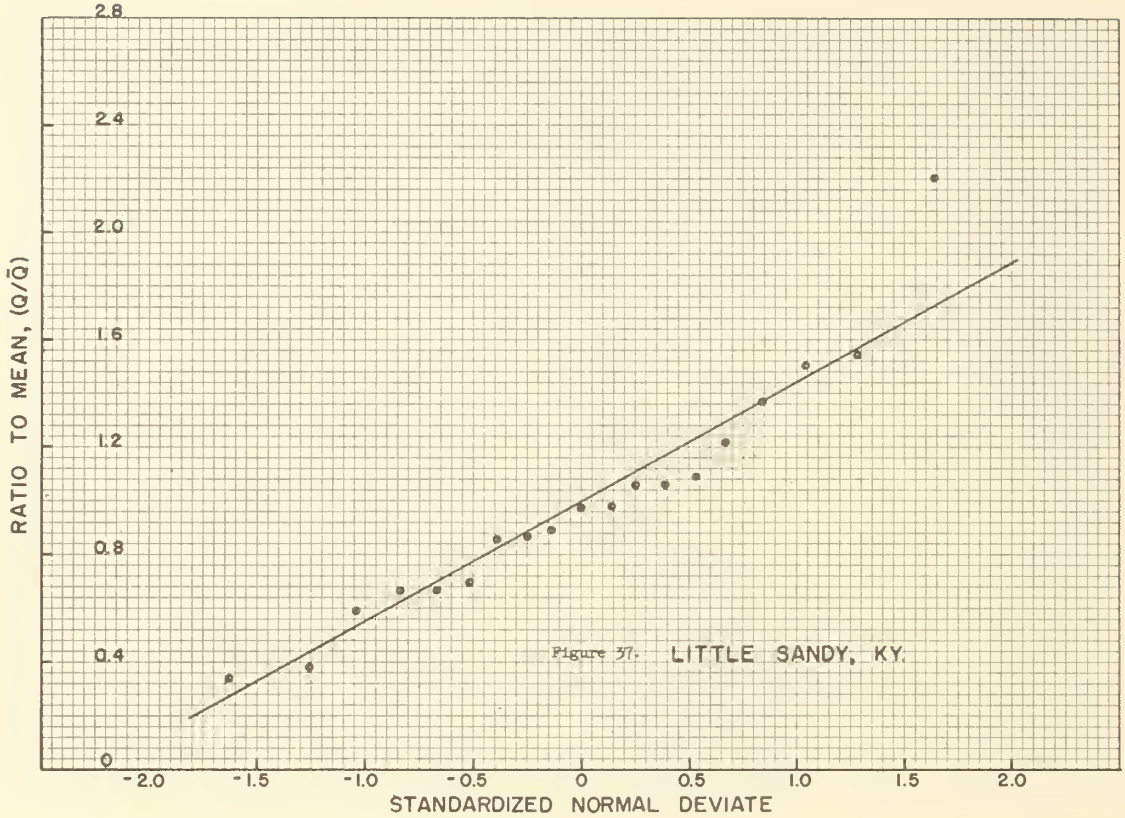


Figure 37. LITTLE SANDY, KY.

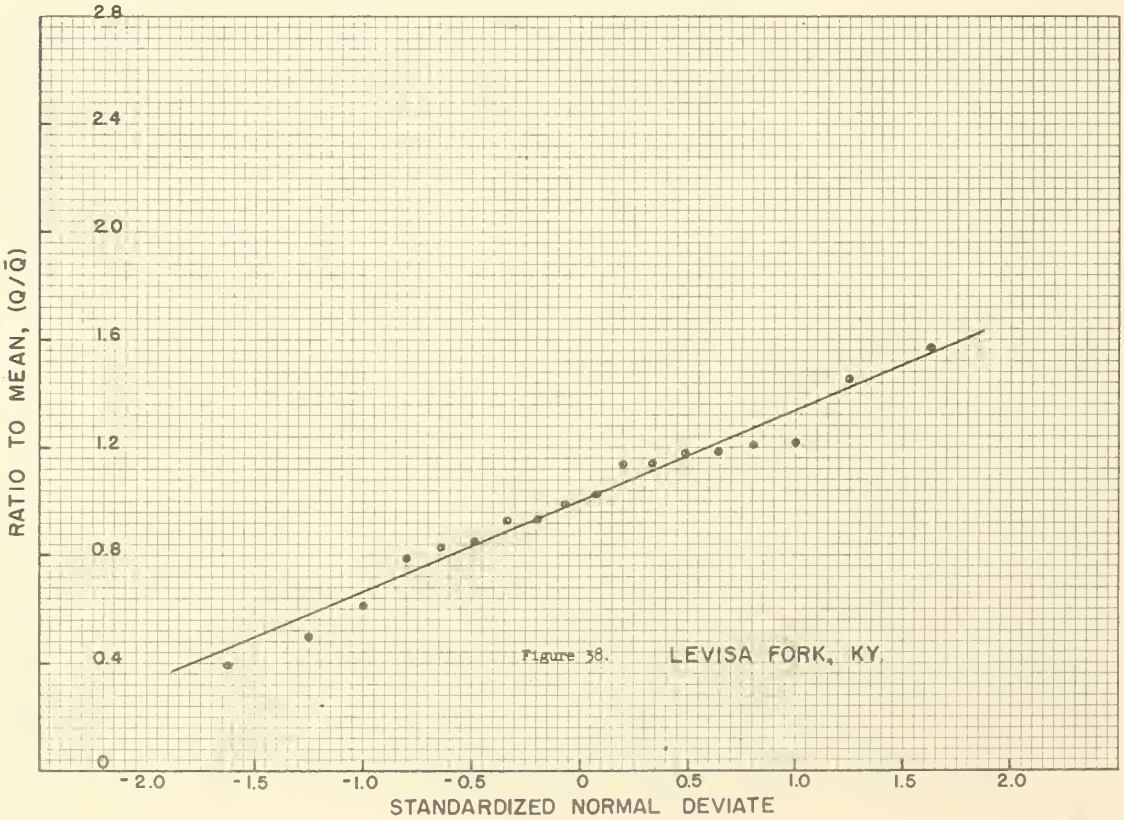


Figure 38. LEVISA FORK, KY.

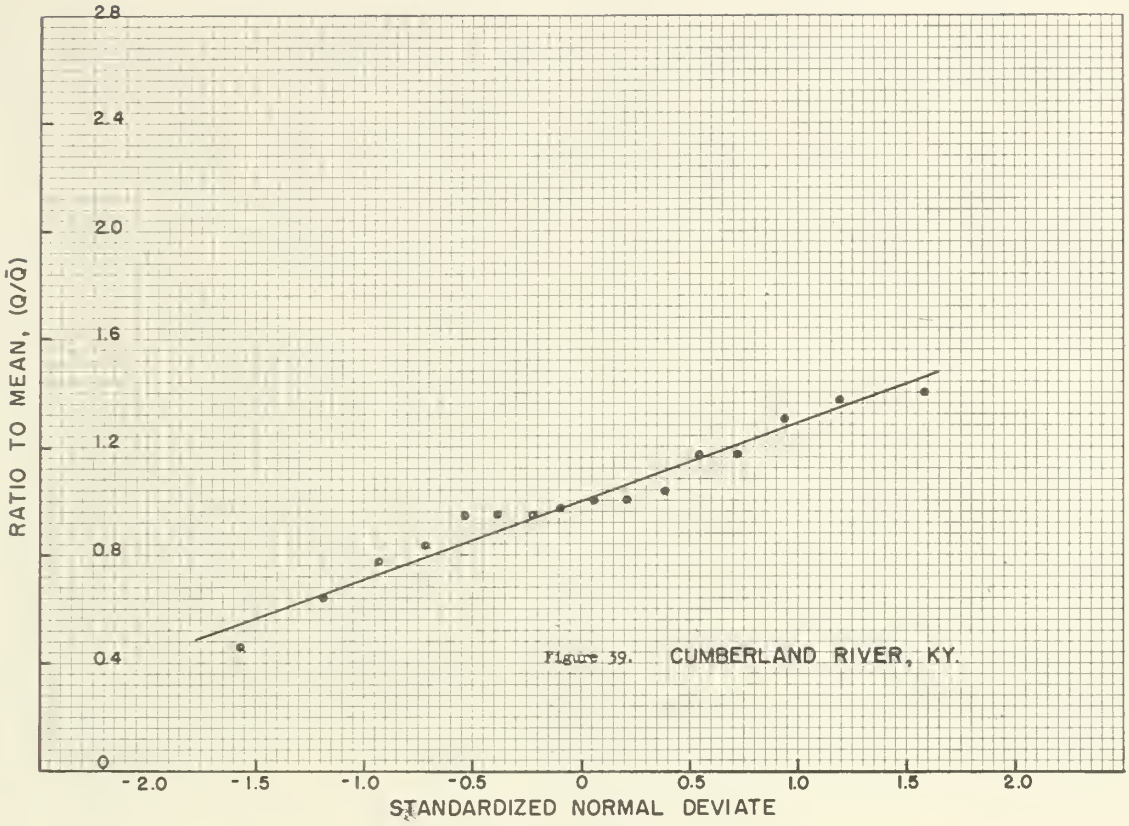


Figure 39. CUMBERLAND RIVER, KY.

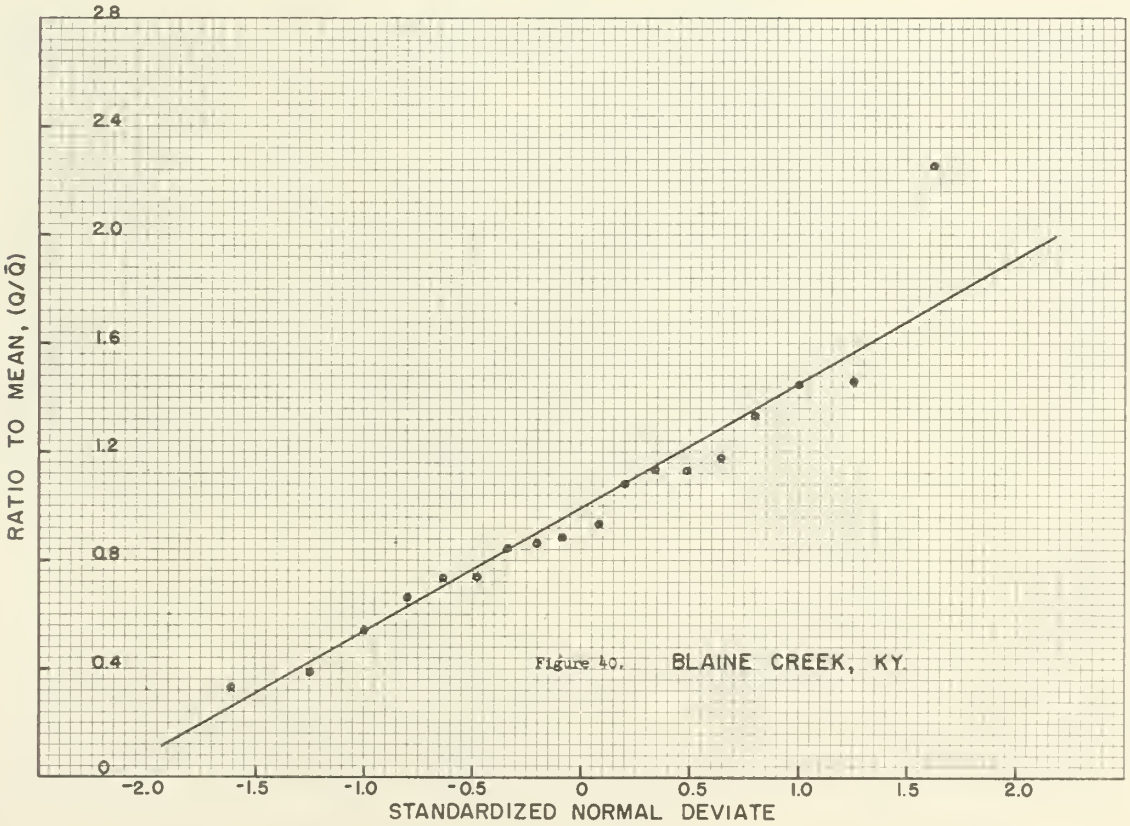


Figure 40. BLAINE CREEK, KY.

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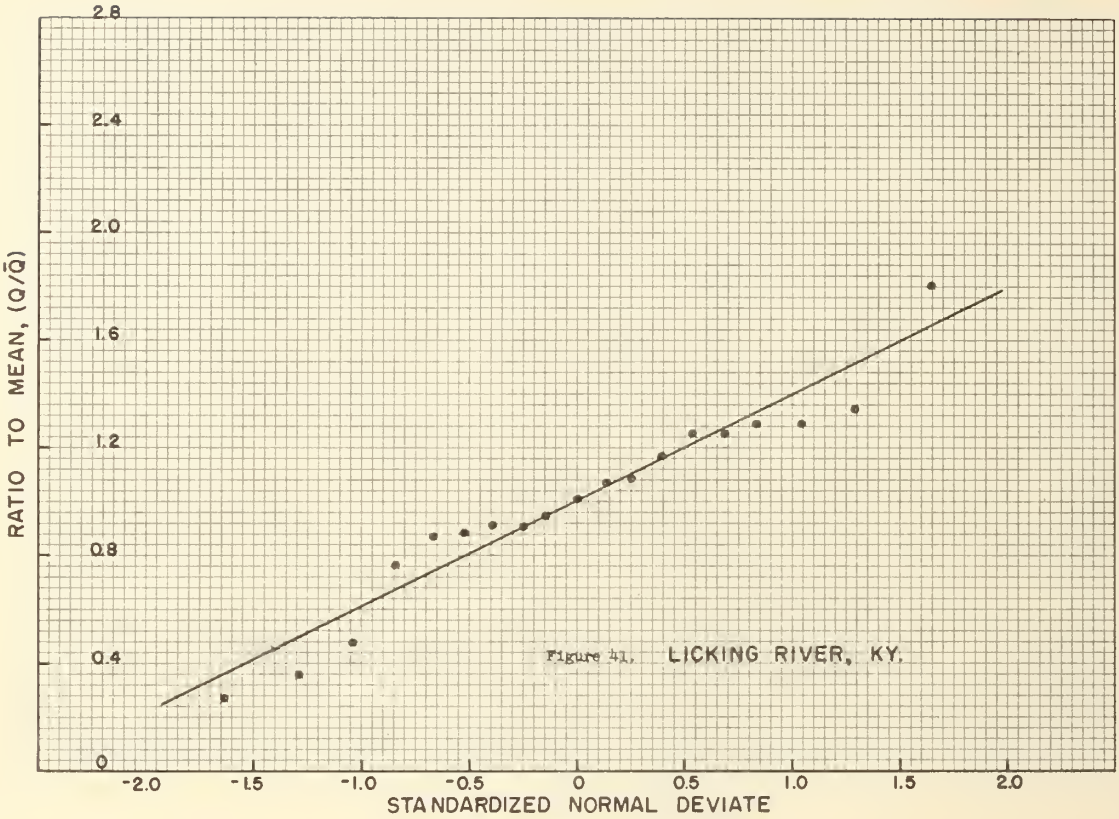

Figure 41. LICKING RIVER, KY.

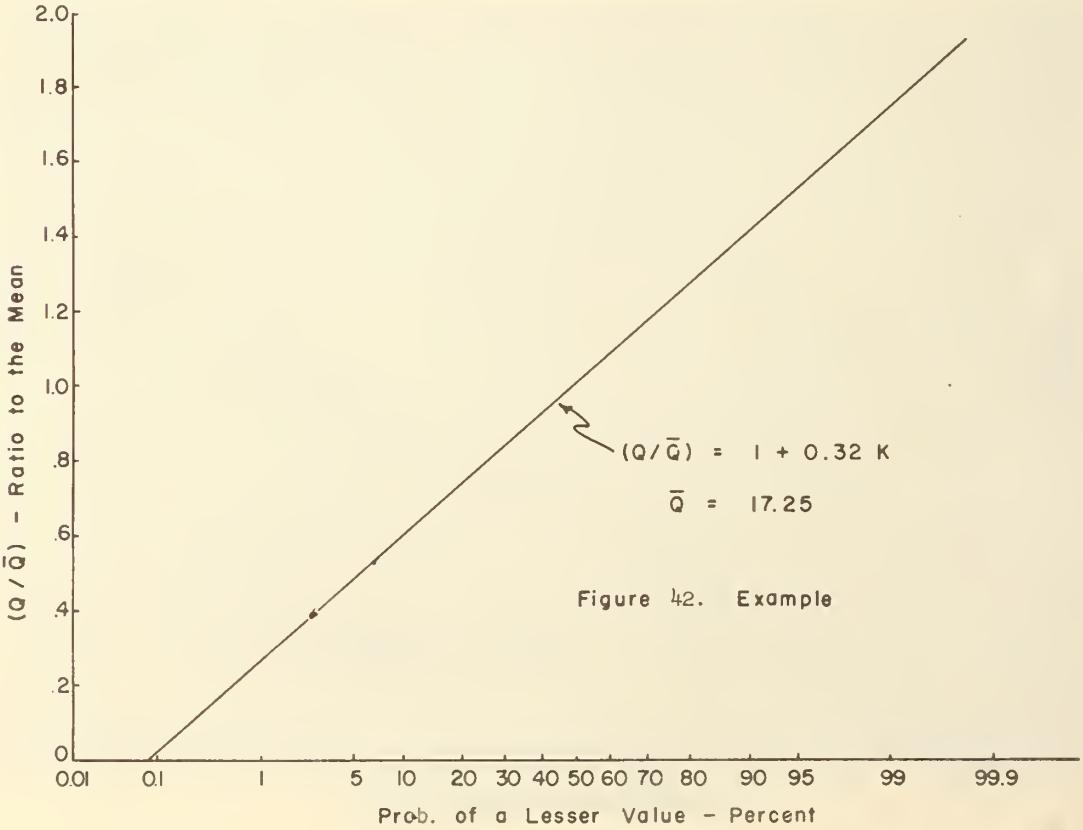

Figure 42. Example